FINAL GRADUATION PROJECT MONOGRAPH

# On the Performance of Fluid Antennas Systems under $\alpha$-$\mu$ Fading Channels

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Graduate Course in Communications Network Engineering

DEPARTMENT OF ELECTRICAL ENGINEERING

**UNIVERSITY OF BRASÍLIA**
FACULTY OF TECHNOLOGY

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*Final graduation project monograph submitted to the Department of Electrical Engineering as partial requirement for the degree of Bacharel in Communication Networks Engineering*

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*Dedico este trabalho à minha família e aos amigos que construí ao longo dessa trajetória.*

*Dedico este trabalho à todos que acreditaram em mim durante está trajetória.*

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## RESUMO

Neste estudo, estatísticas de primeira e segunda ordem são deduzidas para sistemas de antenas fluidas (FAS) sob canais sujeitos ao desvanecimento $\alpha$-$\mu$. Com base nisso e para avaliar o desempenho dos sistemas mencionados, expressões também são derivadas para a probabilidade de indisponibilidade (OP) e capacidade ergódica do canal. Além disso, uma redução exata da OP devido à $N$-ésima porta para uma $(N-1)$ porta também é exibida. Todas as expressões obtidas deste estudo são novas. Várias curvas são mostradas sob diferentes valores dos parâmetros que caracterizam o sistema e a não linearidade do canal. Este é o primeiro trabalho em que o efeito da não linearidade do canal é evidenciado em FAS.

**Palavras-chave: Sistemas de antenas fluidas, capacidade ergódica do canal, estatísticas de primeira e segunda ordem, probabilidade de indisponibilidade.**

## ABSTRACT

In this study, first and second order statistics are deduced for fluid antennas systems (FAS) under $\alpha$-$\mu$ fading channels. Based on this and in order to evaluate the performance of the mentioned systems, expressions are also deduced for the outage probability (OP) and ergodic channel capacity. Furthermore, an exact reduction of the OP due to $N$-th port for an $(N-1)$-port is also presented. All expressions derived in this study are new. Several curves are shown under different values for the system parameters and channel non-linearity. This is the first work in which the effect of the channel non-linearity is evidenced in a FAS.

**Keywords: Fluid antenna systems, ergodic channel capacity, first and second order statistics, outage probability.**

# SUMMARY

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# List of Abbreviations and Acronyms

## Acronyms

| | |
|---|---|
| AFD | *Average Fade Duration* |
| CDF | *Cumulative Distribution Function* |
| FAS | *Fluid Antenna System* |
| LCR | *Level Crossing Rate* |
| MIMO | *Multiple-Input Multiple-Output* |
| MRC | *Maximum-Ratio Combining* |
| OP | *Outage Probability* |
| PDF | *Probability Density Function* |
| SNR | *Signal-to-Noise Ratio* |

# 1 INTRODUCTION

This chapter presents the state-of-the-art of the topic in study; the main contributions obtained by us as well as the organization of this work.

## 1.1 OVERVIEW

Fluid antenna system (FAS) is an emerging topic and has been studied by several researchers over the past years [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11], with potential application in the new emerging wireless technologies and being, recently, proposed as a possible solution to overcome the physical limitation of space present in multiple-input multiple-output (MIMO) systems [3]. The existing literature presents various works on fluid antennas, shedding light on their capabilities and potential applications.

In [1], the concept of FAS is discussed. Wong *et. al.* [1] devised a system in which a single antenna can change its position instantly in a linear space and named it FAS. In the aforementioned work, the receiving antenna is isotropic and its location can be switched between one of the $N$ possible predefined locations, also called ports. In turn, the performance limits of the mentioned systems are introduced in [2], in which expressions are presented for the level crossing rate (LCR), the average fade duration (AFD) and ergodic channel capacity for the $N$-port FAS. In addition, a closed-form capacity lower bound is also presented in [2]. Expressions for the probability density function (PDF) and cumulative distribution function (CDF) of the envelopes at all the ports for the FAS are deduced in [3], considering correlated Rayleigh fading channels. An exact, approximated and upper bound expressions are derived for the outage probability (OP).

FAS's are studied in [4] over correlated Nakagami-$m$ fading channels. New expressions for the PDF and CDF are derived by the authors and the performance of the mentioned system is also assessed by OP. Closed-form expressions are deduced in [5] in order to characterize the LCR, considering the practical constraints and limitations. In [2, 3, 4, 5], it should be noted that the same correlation model between the ports is considered, which has the disadvantage of requiring a reference port. In order to solve this problem, a more realistic model for correlation is presented in [13], in which there is no need for a reference port (*i.e.*, any port is a reference to another port). Recently, FAS has been studied in different contexts and scenarios, such as multiple access [6, 7], large-scale cellular networks [8] and MIMO evolution beyond 5G through reconfigurable intelligent surfaces [9]. In [10, 11], the problem of port selection for FAS is investigated.

In a literature review, it is observed that all available works on FAS consider simple fading models, such as Rayleigh or Nakagami-$m$. However, the mentioned models characterize only the multipath effect. Over the years, several studies have explained that wireless communication channels can also be affected by the non linearity of the propagation medium. Thus, not incorporating the effect of nonlinearity in the models is not a realistic assumption.

In this work and for the first time, the performance of FAS under $\alpha$-$\mu$ fading channels is evaluated.

Several new expressions for the first and second order statistics are deduced and used to derive metrics for evaluate the mentioned systems. In our study, as mentioned, the $\alpha$-$\mu$ distribution is adopted to characterize the small-scale fading. The $\alpha$-$\mu$ channel model [12] has been extensively supported by experimental results in the technical literature. Furthermore, it is written in terms of physical parameters and adopted in many works. In addition, the $\alpha$-$\mu$ jointly considers the multipath fading and the non-linearity of the propagation medium, that makes this distribution able to model realistic environments [12]. As the $\alpha$-$\mu$ encompass other models as special cases, it should be mentioned that this letter is connected with other works, such as [2, 3, 4], in which some results can be obtained as a particular case of the study presented in this work.

## 1.2 CONTRIBUTIONS OF THIS WORK

This is the first work in which the impact of the non-linearity of the propagation medium, modeled by $\alpha$-$\mu$ distribution, is studied in FAS systems. Our work open new fronts for further investigations, since we provide analytical and simulation results to reveal how the channel non-linearity affects FAS performance. The main contributions of this studies are:

- New expressions are derived for the first order statistics, such as PDF and CDF; and for the second order statistics, such as LCR; of the envelopes at all the ports.
- New expressions are derived for OP and ergodic channel capacity.
- An exact reduction of the OP due to $N$-th port for an $(N-1)$-port are also presented.
- An evaluation of the impact of the correlation coefficient between the ports on the performance of FAS is presented.

## 1.3 PAPERS SUBMITTED

- P. D. Alvim, F. O. Barcelos, H. S. Silva, U. S. Dias and R. A. A. de Souza, "On the Performance of Fluid Antennas Systems under $\alpha$-$\mu$ Fading Channels", under review in *IEEE Wireless Communications Letters*, Jun. 2023.

## 1.4 ORGANIZATION OF THIS WORK

The remaining of the work is organized as follows. Chapter 2 describes the system and channel models adopted. Expressions for important statistics of FAS under $\alpha$-$\mu$ fading channels are presented in Chapter 3. Metrics are presented in Chapter 4. Chapter 5 shows the numerical results and discussions. Chapter 6 brings the conclusions of the study.

# 2 SYSTEM AND CHANNEL MODELS

In this chapter, the system, channel and coefficient correlation models considered in our work are presented.

## 2.1 FLUID ANTENNAS SYSTEMS

A FAS is considered in this work, based on [3]. In the mentioned system, there are $N$ different fixed locations for the best reception of the signal, distributed over a linear dimension $W\lambda$, in which $\lambda$ is the wavelength and $W > 0$ is a constant related to the size of the FAS, as see in Fig 2.1. It should be mentioned that each location is referred as port and an antenna at location port $k$ is considered as an ideal point antenna.

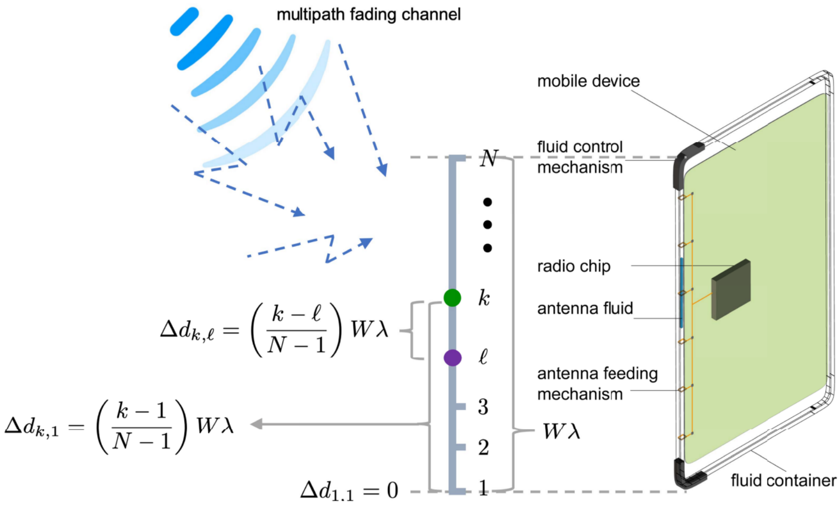



Figure 2.1: A possible architecture for a FAS [3, see Fig. 2].

## 2.2 CORRELATION MODELS

In this work, the mathematical models for the correlation between the ports ($\delta_k$) presented in [2] and [13] are considered. In [2], the first port is the reference location and the other $N-1$ ports are correlated to the first one. In a FAS, the correlation effect occurs since the ports are close to each other. The power correlation coefficient, denoted by $\delta_k$, is written as

$$\delta_k = J_0^2\left(\frac{2\pi(k-1)W}{N-1}\right), \; k = 2, \ldots, N, \tag{2.1}$$

In (2.1), $J_0(\cdot)$ is the zero-order Bessel function of the first kind.

In this study, a realistic power correlation coefficient model is also considered. In [13], a common correlation coefficient ($\delta = \delta_k$) between all ports is adopted, in which there is no need for a reference port (i.e., any port is a reference to another port). The power correlation coefficient $\delta$ is written as [13, Eq. (5)]

$$\delta = 2\left[{}_1\mathrm{F}_2\left(\frac{1}{2};1;\frac{3}{2};-\pi^2W^2\right) - \frac{J_1(2\pi W)}{2\pi W}\right], \forall\, k, \tag{2.2}$$

where $J_1(\cdot)$ is the first-order Bessel function of the first kind [14, id 03.01.02.0001.01] and ${}_\mathrm{a}F_\mathrm{b}(\cdot;\cdot;\cdot)$ represents the generalized hypergeometric function.

## 2.3 RECEIVED SIGNAL AND FADING MODELS

The received signal at the $k$-th port in a FAS is given by [3, Eq. (3)]

$$y_k = g_k x + \eta_k, \tag{2.3}$$

in which $x$ is the transmitted signal, $\eta_k$ is the zero-mean $\_\sigma_\eta^2$-variance complex additive white Gaussian noise at the $k$-th port, and $g_k$ is the complex envelope fading, where $h_k \triangleq |g_k|$ is the normalized envelope, modeled in this study by the $\alpha$-$\mu$ distribution, whose PDF is given by [12, Eq. (3)]

$$f_{h_k}(\rho_k) = \frac{\alpha_k \mu^\mu \rho_k^{\alpha_k\mu-1}}{\Gamma(\mu)\exp(\mu\rho_k^{\alpha_k})}, \quad \rho_k \geq 0. \tag{2.4}$$

In (2.4), $\Gamma(\cdot)$ is the Gamma function [14, id. 06.05.02.0001.01], $\alpha$ characterizes the non-linearity of the propagation medium, and $\mu$ represents the number of multipath clusters. In a fluid antenna system, it is assumed that the port with the strongest channel condition is always selected, i.e., [5, Eq. (3)]

$$h = \max\{h_1, h_2, \cdots, h_N\}. \tag{2.5}$$

In our study, the $\alpha$-$\mu$ distribution is adopted since it is generalist, flexible, and easy to manipulate mathematically. The mentioned fading model considers a signal consisting of multipath clusters and explores the non linearity of the propagation medium. This make the $\alpha$-$\mu$ distribution very attractive to be used in FAS. The $\alpha$-$\mu$ distribution encompasses a lot of models presented in the literature. In fact, by properly selecting the fading parameters $\alpha$ and $\mu$, the models presented in Table 2.1 can be obtained.

Table 2.1: Special cases.

| Fading Models | Parameters |
|---|---|
| Rayleigh | $\alpha = 2$, $\mu = 1$ |
| Nakagami-$m$ | $\alpha = 2$, $\mu = m$ |
| Weibull | $\alpha = \alpha$, $\mu = 1$ |
| One-Gaussian | $\alpha = 2$, $\mu = 0.5$ |

# 3 FIRST AND SECOND ORDER STATISTICS OF FAS UNDER $\alpha - \mu$ FADING CHANNELS

In this chapter, important statistics are deduced, such as PDFs, CDF, and LCR. The first order statistics are used in this work in order to derive performance metrics to evaluate the impact of the channel parameters in the FAS. Furthermore, the LCR is a key second-order statistic used to provide useful information about the dynamic temporal behavior of multipath fading channels. LCR is a statistical measure of how often a random signal crosses a certain threshold. The expressions derived by us are valid for any arbitrary correlation values.

## 3.1 CONDITIONAL PDF OF $H_2$ GIVEN $H_1$

**Proposition 3.1.1.** *Let $h_2$ and $h_1 \in \mathbb{R}^+$. The conditional PDF of $h_2$ given $h_1$, denoted by $f_{h_2|h_1}(\rho_2|\rho_1)$, is written as*

$$f_{h_2|h_1}(\rho_2|\rho_1) = \frac{\mu \rho_1^{-\frac{\alpha_1}{2}(\mu-1)} \rho_2^{\frac{\alpha_2}{2}(\mu+1)-1} \alpha_2}{(1-\delta_2)\delta_2^{\frac{\mu-1}{2}}} \exp\left[-\mu\left(\frac{\rho_2^{\alpha_2}+\delta_2\rho_1^{\alpha_1}}{1-\delta_2}\right)\right] \mathrm{I}_{\mu-1}\left(\frac{2\mu\sqrt{\delta_2\rho_1^{\alpha_1}\rho_2^{\alpha_2}}}{1-\delta_2}\right), \quad (3.1)$$

*in which $\delta_2$ is the power correlation coefficient of the second port with respect to the first one and $\mathrm{I}_v(\cdot)$ is the modified Bessel function of first kind and order $v$ [14, id 03.02.02.0001.01].*

*Proof.* The PDF $f_{h_2|h_1}(\rho_2|\rho_1)$ is computed by taking a ratio of $f_{h_2,h_1}(\rho_2,\rho_1)$ over the normalized PDF of $h_1$ as

$$f_{h_2|h_1}(\rho_2|\rho_1) = \frac{f_{h_2,h_1}(\rho_1,\rho_2)}{f_{h_1}(\rho_1)}, \quad (3.2)$$

in which the joint PDF of two correlated $\alpha$-$\mu$ is given by [12, Eq. (28)]

$$f_{h_2,h_1}(\rho_1,\rho_2) = \frac{\alpha_1\alpha_2\mu^{\mu+1}\rho_1^{\frac{\alpha_1}{2}(\mu+1)-1}\rho_2^{\frac{\alpha_2}{2}(\mu+1)-1}}{(1-\delta_2)\delta_2^{\frac{\mu-1}{2}}\Gamma(\mu)} \exp\left(-\mu\frac{\rho_1^{\alpha_1}+\rho_2^{\alpha_2}}{1-\delta_2}\right)\mathrm{I}_{\mu-1}\left(\frac{2\mu\sqrt{\delta_2\rho_1^{\alpha_1}\rho_2^{\alpha_2}}}{1-\delta_2}\right). \quad (3.3)$$

Substituting (3.3) and (2.4) into $f_{h_2|h_1}(\rho_2|\rho_1)$, (3.1) can be obtained after simplifications. Hence, the proof is complete.

□

## 3.2 THE JOINT PDF OF $N$ CORRELATED $\alpha$-$\mu$ RANDOM VARIATES

**Proposition 3.2.1.** *Let $h_k \in \mathbb{R}^+$, with $k = 1, 2, \ldots, N$. The joint PDF of $N$ correlated $\alpha$-$\mu$ random variates (RV) is denoted by $f_{h_1,h_2,\ldots,h_N}(\rho_1, \rho_2, \ldots, \rho_N)$ and written as*

$$f_{h_1,h_2,\ldots,h_N}(\rho_1, \rho_2, \ldots, \rho_N) = \frac{\alpha_1 \mu^\mu \rho_1^{\alpha_1\mu-1}}{\Gamma(\mu)\exp\left(\mu\rho_1^{\alpha_1}\right)} \prod_{k=2}^{N} \frac{\mu \rho_1^{-\frac{\alpha_1}{2}(\mu-1)} \rho_k^{\frac{\alpha_k}{2}(\mu+1)-1} \alpha_k}{(1-\delta_k)\delta_k^{\frac{\mu-1}{2}}} \times \exp\left[-\mu\left(\frac{\rho_k^{\alpha_k} + \delta_k \rho_1^{\alpha_1}}{1-\delta_k}\right)\right] \mathrm{I}_{\mu-1}\left(\frac{2\mu\sqrt{\delta_k \rho_1^{\alpha_1}\rho_k^{\alpha_k}}}{1-\delta_k}\right). \tag{3.4}$$

*Proof.* The joint PDF of $N$ correlated $\alpha$-$\mu$ RV, $f_{h_1,h_2,\ldots,h_N}(\rho_1, \rho_2, \ldots, \rho_N)$, can be written as

$$f_{h_1,h_2,\ldots,h_N}(\rho_1, \rho_2, \ldots, \rho_N) = f_{h_1}(\rho_1) f_{h_2,\ldots,h_N|h_1}(\rho_2, \ldots, \rho_N|\rho_1). \tag{3.5}$$

Since the independence between the ports conditioned on port 1, it follows that

$$f_{h_2,\ldots,h_N|h_1}(\rho_2, \ldots, \rho_N|\rho_1) = \prod_{k=2}^{N} f_{h_k|h_1}(\rho_k|\rho_1). \tag{3.6}$$

Using (3.1), (3.6) can be written as

$$f_{h_2,\ldots,h_N|h_1}(\rho_2, \ldots, \rho_N|\rho_1) = \prod_{k=2}^{N} \frac{\mu \rho_1^{-\frac{\alpha_1}{2}(\mu-1)} \rho_k^{\frac{\alpha_k}{2}(\mu+1)-1} \alpha_k}{(1-\delta_k)\delta_k^{\frac{\mu-1}{2}}} \times \exp\left[-\mu\left(\frac{\rho_k^{\alpha_k} + \delta_k \rho_1^{\alpha_1}}{1-\delta_k}\right)\right] \mathrm{I}_{\mu-1}\left(\frac{2\mu\sqrt{\delta_k \rho_1^{\alpha_1}\rho_k^{\alpha_k}}}{1-\delta_k}\right). \tag{3.7}$$

Substituting (3.7) and (2.4) in (3.5), (3.4) is deduced, that complete the proof. □

**Corollary 3.2.1.1.** *For the case in which no correlation is considered, i.e. $\delta_k$ equals zero, it follows that*

$$f_{h_1,h_2,\ldots,h_N}(\rho_1, \rho_2, \ldots, \rho_N) = \prod_{k=1}^{N} f_{h_k}(\rho_k). \tag{3.8}$$

*Proof.* Making $\delta_k \to 0$ in (3.4), using the fact that

$$\lim_{\delta_k \to 0} \frac{\exp\left[-\mu\left(\frac{\delta_k \rho_1^{\alpha_1} + \rho_k^{\alpha_k}}{1-\delta_k}\right)\right]}{(1-\delta_k)\delta_k^{\frac{\mu-1}{2}}} \mathrm{I}_{\mu-1}\left(\frac{2\mu\sqrt{\delta_k \rho_1^{\alpha_1}\rho_k^{\alpha_k}}}{1-\delta_k}\right) = \frac{\exp\left[-\mu\rho_k^{\alpha_k}\right]}{\Gamma(\mu)} \left(\mu\sqrt{\rho_1^{\alpha_1}\rho_k^{\alpha_k}}\right)^{\mu-1} \tag{3.9}$$

and, in sequence, utilizing the result above again in (3.4), (3.8) is obtained after simplifications. Hence, the proof is complete. □

## 3.3 THE CONDITIONAL JOINT CDF

**Proposition 3.3.1.** *The joint CDF of* $h_1, h_2, \ldots, h_N$*, denoted by* $F_{h_1,h_2,\ldots,h_N}(\rho_1, \rho_2, \ldots, \rho_N)$*, is given by*

$$F_{h_1,h_2,\ldots,h_N}(\rho_1,\rho_2,\ldots,\rho_N)=\frac{\alpha_1\mu^\mu}{\Gamma(\mu)}\int_0^{\rho_1} t_1^{\alpha_1\mu-1}\exp\left(-\mu t_1^{\alpha_1}\right) \times\prod_{k=2}^{N}\left[1-\mathrm{Q}_\mu\left(\sqrt{\frac{2\mu\delta_k t_1^{\alpha_1}}{1-\delta_k}},\sqrt{\frac{2\mu\rho_k^{\alpha_k}}{1-\delta_k}}\right)\right]\mathrm{d}t_1, \tag{3.10}$$

*in which* $\mathrm{Q}_\mu(\cdot,\cdot)$ *is the Marcum Q-function.*

*Proof.* The joint CDF of $h_1, h_2, \cdots, h_N$ can be evaluated by means of

$$F_{h_1,h_2,\ldots,h_N}(\rho_1,\rho_2,\ldots,\rho_N)=\int_0^{\rho_1}\cdots\int_0^{\rho_N} f_{h_1,h_2,\ldots,h_N}(t_1,t_2,\ldots,t_N)\mathrm{d}t_1\cdots\mathrm{d}t_N. \tag{3.11}$$

Replacing (3.4) in $F_{h_1,h_2,\ldots,h_N}(\rho_1,\rho_2,\ldots,\rho_N)$, using [15, Eq. (8.445)] and proceeding with some simplifications, it follows that (3.12) can be obtained.

$$F_{h_1,h_2,\ldots,h_N}(\rho_1,\rho_2,\ldots,\rho_N)= \frac{\alpha_1\mu^\mu}{\Gamma(\mu)}\prod_{k=2}^{N}\frac{\mu^\mu\alpha_k}{(1-\delta_k)^\mu}\int_0^{\rho_1}\frac{t_1^{\alpha_1\mu-1}}{\exp(\mu t_1^{\alpha_1})}\exp\left(-\sum_{k=2}^{N}\mu t_1^{\alpha_1}\left(\frac{1}{1-\delta_k}-1\right)\right) \times\left[\prod_{k=2}^{N}\sum_{i_k=0}^{\infty}\frac{1}{i_k!\Gamma(\mu+i_k)}\left(\frac{\mu^2\delta_k t_1^{\alpha_1}}{(1-\delta_k)^2}\right)^{i_k}\int_0^{\rho_k} t_k^{\alpha_k(\mu+i_k)-1}\exp\left(\frac{-\mu t_k^{\alpha_k}}{1-\delta_k}\right)\mathrm{d}t_k\right]\mathrm{d}t_1. \tag{3.12}$$

Applying the variable change $x = t_k^{\alpha_k}$ and using [15, Eq. (3.381.1)], the inner integral of (3.12) with respect of $t_k$ can be solved. After simplifications,

$$F_{h_1,h_2,\ldots,h_N}(\rho_1,\rho_2,\ldots,\rho_N)= \frac{\alpha_1\mu^\mu}{\Gamma(\mu)}\int_0^{\rho_1}\frac{t_1^{\alpha_1\mu-1}}{\exp(\mu t_1^{\alpha_1})}\exp\left(-\sum_{k=2}^{N}\mu t_1^{\alpha_1}\left(\frac{1}{1-\delta_k}-1\right)\right)\left[\prod_{k=2}^{N}\sum_{i_k=0}^{\infty}\frac{(\mu\delta_k t_1^{\alpha_1})^{i_k}\gamma\left(\mu+i_k,\frac{\mu\rho_k^{\alpha_k}}{1-\delta_k}\right)}{(1-\delta_k)^{i_k}\Gamma(\mu+i_k)i_k!}\right]\mathrm{d}t_1. \tag{3.13}$$

Using [15, Eq. (8.356.3)] and [16, Eq. (9)]

$$\mathrm{Q}_\mu(A,\beta)=\exp\left(-\frac{A^2}{2}\right)\sum_{i_k=0}^{\infty}\frac{1}{i_k!}\left(\frac{A^2}{2}\right)^{i_k}\frac{\Gamma(\mu+i_k,\beta^2/2)}{\Gamma(\mu+i_k)}, \tag{3.14}$$

then (3.10) is deduced after simplifications, with

$$A = \sqrt{\frac{2\mu\delta_k\rho_1^{\alpha_1}}{1-\delta_k}} \tag{3.15}$$

and

$$\beta = \sqrt{\frac{2\mu P_k^{\alpha_k}}{1-\delta_k}}. \tag{3.16}$$

Hence, the proof is complete. □

## 3.4 LEVEL CROSSING RATE

**Proposition 3.4.1.** *The LCR, denoted by $L(\rho_{\rm th})$, is the measurement of the average number of times at which the envelope $h$ crosses a certain threshold level $\rho_{\rm th}$ and, for a $N$-port FAS under $\alpha$-$\mu$ fading channels, is given by (3.17) with $f_{\rm D}$ denoting the maximum Doppler frequency and $j = 1, 2, \cdots, N$.*

$$\begin{aligned} L(\rho_{\rm th}) = & \frac{\sqrt{2\pi} f_{\rm D}\Gamma\left(\mu - \frac{1}{2} + \frac{1}{\alpha_j}\right)}{\alpha_j \mu^{\frac{1}{\alpha_j}}\Gamma(\mu)} \frac{\mu^\mu \alpha_1}{\rho_{\rm th}\Gamma(\mu)} \\ & \left\{ \frac{\rho_{\rm th}^{\alpha_1\mu}}{\exp\left(\mu\rho_{\rm th}^{\alpha_1}\right)} \prod_{k=2}^{N}\left[1 - {\rm Q}_\mu\left(\sqrt{\frac{2\mu\delta_k\rho_{\rm th}^{\alpha_1}}{1-\delta_k}}, \sqrt{\frac{2\mu\rho_{\rm th}^{\alpha_k}}{1-\delta_k}}\right)\right] + \sum_{i=2}^{N} \frac{\alpha_i\mu\rho_{\rm th}^{\frac{\alpha_i}{2}(\mu+1)}}{(1-\delta_i)\delta_i^{\frac{\mu-1}{2}}\exp\left(\frac{\mu\rho_{\rm th}^{\alpha_i}}{1-\delta_i}\right)} \right. \\ & \left. \int_0^{\rho_{\rm th}} \frac{\rho_1^{\frac{\alpha_1}{2}(\mu+1)-1}}{\exp\left(\frac{\mu\rho_1^{\alpha_1}}{1-\delta_i}\right)} {\rm I}_{\mu-1}\left(\frac{2\mu\sqrt{\delta_i\rho_1^{\alpha_1}\rho_{\rm th}^{\alpha_i}}}{1-\delta_i}\right) \prod_{\substack{k=2\\k\neq i}}^{N}\left[1 - {\rm Q}_\mu\left(\sqrt{\frac{2\mu\delta_k\rho_1^{\alpha_1}}{1-\delta_k}}, \sqrt{\frac{2\mu\rho_{\rm th}^{\alpha_k}}{1-\delta_k}}\right)\right]{\rm d}\rho_1 \right\} \end{aligned} \tag{3.17}$$

*Proof.* The LCR is mathematically given by $L(\rho_{\rm th}) = \int_0^\infty \dot{\rho} f_{\dot{h},h}(\dot{\rho}, \rho_{\rm th}){\rm d}\dot{\rho}$, in which $\dot{\rho}$ is the time derivative of $\rho$ and $f_{\dot{h},h}(\cdot,\cdot)$ is the joint PDF of $h$ and $\dot{h}$. The LCR $L(\rho_{\rm th})$ can be rewritten as presented in [5, Eqs. (13) and (14)], that is composed by two terms multiplied by [5, Eq. (12)]

$$\int_0^\infty \dot{\rho} f_{\dot{h}_i}(\dot{\rho}){\rm d}\dot{\rho} = \frac{\sqrt{2\pi} f_{\rm D}\Gamma\left(\mu - \frac{1}{2} + \frac{1}{\alpha_j}\right)}{\alpha_j \mu^{\frac{1}{\alpha_j}}\Gamma(\mu)}, \ \forall j = 1, \cdots, N. \tag{3.18}$$

For the first term of [5, Eq. (14)], it follows that

$$\underbrace{\int_0^{\rho_{\rm th}}\cdots\int_0^{\rho_{\rm th}}}_{(N-1)-\text{fold}} f_{h_1,h_2,\ldots,h_N}(\rho_1=\rho_{\rm th},\rho_2,\ldots,\rho_N)\underbrace{{\rm d}\rho_2\cdot{\rm d}\rho_N}_{(N-1)-\text{fold}}$$

$$\overset{(a)}{=}\frac{\alpha_1\mu^\mu\rho_{\rm th}^{\alpha_1\mu-1}}{\Gamma(\mu)\exp(\mu\rho_{\rm th}^{\alpha_1})}\prod_{k=2}^{N}\int_0^{\rho_{\rm th}}\frac{\mu\rho_{\rm th}^{-\frac{\alpha_1}{2}(\mu-1)}\rho_k^{\frac{\alpha_k}{2}(\mu+1)-1}\alpha_k}{(1-\delta_k)\delta_k^{\frac{\mu-1}{2}}}$$

$$\times\exp\left[\frac{\mu\rho_{\rm th}^{\alpha_1}\delta_k+\mu\rho_k^{\alpha_k}}{1-\delta_k}\right]{\rm I}_{\mu-1}\left(\frac{2\mu\sqrt{\delta_k\rho_{\rm th}^{\alpha_1}\rho_k^{\alpha_k}}}{1-\delta_k}\right){\rm d}\rho_k$$

$$\overset{(b)}{=}\frac{\alpha_1\mu^\mu\rho_{\rm th}^{\alpha_1\mu-1}}{\Gamma(\mu)\exp\left(\mu\rho_{\rm th}^{\alpha_1}\right)}\prod_{k=2}^{N}\left[1-{\rm Q}_\mu\left(\sqrt{\frac{2\mu\delta_k\rho_{\rm th}^{\alpha_1}}{1-\delta_k}},\sqrt{\frac{2\mu\rho_{\rm th}^{\alpha_k}}{1-\delta_k}}\right)\right], \tag{3.19}$$

where (a) is deduced using (3.5), (2.4), (3.7) and (b) follows from [15, Eq. (8.356.3)] and [16, Eq. (9)]. The second term is given by (3.20), derived making some variables changes and using [16, Eq. (1)].

$$\sum_{i=2}^{N}\underbrace{\int_0^{\rho_{\rm th}}\cdots\int_0^{\rho_{\rm th}}}_{(N-1)-\text{fold}} f_{h_1,h_2,\ldots,h_N}(\rho_1,\ldots,\rho_i=\rho_{\rm th},\ldots,\rho_N)\ \underbrace{{\rm d}\rho_1\cdots{\rm d}\rho_N}_{(N-1)-\text{fold},\ k\neq i}$$

$$=\sum_{i=2}^{N}\frac{\alpha_i\mu^{\mu+1}\rho_{\rm th}^{\frac{\alpha_i}{2}(\mu+1)-1}}{(1-\delta_i)\delta_i^{\frac{\mu-1}{2}}\exp\left(\frac{\mu\rho_{\rm th}^{\alpha_i}}{1-\delta_i}\right)}$$

$$\times\int_0^{\rho_{\rm th}}\frac{\alpha_1\rho_1^{\frac{\alpha_1}{2}(\mu+1)-1}}{\Gamma(\mu)\exp\left(\frac{\mu\rho_1^{\alpha_1}}{1-\delta_i}\right)}{\rm I}_{\mu-1}\left(\frac{2\mu\sqrt{\delta_i\rho_1^{\alpha_1}\rho_{\rm th}^{\alpha_i}}}{1-\delta_i}\right)\prod_{\substack{k=2\\k\neq i}}^{N}\left[1-{\rm Q}_\mu\left(\sqrt{\frac{2\mu\delta_k\rho_1^{\alpha_1}}{1-\delta_k}},\sqrt{\frac{2\mu\rho_{\rm th}^{\alpha_k}}{1-\delta_k}}\right)\right]{\rm d}\rho_1. \tag{3.20}$$

Replacing (3.18), (3.19) and (3.20) in the previous LCR expression, (3.17) is derived. □

# 4 PERFOMANCE ANALYSIS

In this chapter, metrics such as OP and ergodic channel capacity are presented.

## 4.1 OUTAGE PROBABILITY

The OP is defined as the probability that the received signal power falls below a certain threshold. The OP for a FAS can be derived from the joint CDF given in (3.10) by making $\rho_1{=}\rho_2{=}\cdots{=}\rho_N{=}\sqrt{\gamma_{\mathrm{th}}/\Theta}$, in which $\gamma_{\mathrm{th}}$ is a specified threshold and $\Theta$ is the ratio between $\bar{\gamma}$, the average signal-to-noise ratio (SNR), and $\Omega = \mathrm{E}[h_k^2]$ [12, Eq. (5)]. Thus,

$$P_{\mathrm{out}} = \frac{\alpha_1 \mu^{\mu}}{\Gamma(\mu)} \int_0^{\sqrt{\frac{\Omega\gamma_{\mathrm{th}}}{\bar{\gamma}}}} t_1^{\alpha_1\mu-1} \exp\left(-\mu t_1^{\alpha_1}\right) \prod_{k=2}^{N} \left[1 - \mathrm{Q}_{\mu}\left(\sqrt{\frac{2\mu\delta_k t_1^{\alpha_1}}{1-\delta_k}}, \sqrt{\frac{2\mu}{1-\delta_k}} \left(\frac{\Omega\gamma_{\mathrm{th}}}{\bar{\gamma}}\right)^{\frac{\alpha_k}{2}}\right)\right] \mathrm{d}t_1. \tag{4.1}$$

An exact reduction of the OP, due to $N$-th port for an $(N-1)$-port, can be derived expanding the $N$-th factor in the product present in (4.1). The exact reduction is denoted by $\Delta P_{\mathrm{out}}$ and is written, after simplifications, as

$$\begin{aligned} \Delta P_{\mathrm{out}} = \frac{\alpha_1 \mu^{\mu}}{\Gamma(\mu)} \int_0^{\sqrt{\frac{\Omega\gamma_{\mathrm{th}}}{\bar{\gamma}}}} & t_1^{\alpha_1\mu-1} \exp\left(-\mu t_1^{\alpha_1}\right) \mathrm{Q}_{\mu}\left(\sqrt{\frac{2\mu\delta_N t_1^{\alpha_1}}{1-\delta_N}}, \sqrt{\frac{2\mu}{1-\delta_N}} \left(\frac{\Omega\gamma_{\mathrm{th}}}{\bar{\gamma}}\right)^{\frac{\alpha_N}{2}}\right) \\ & \times \prod_{k=2}^{N-1} \left[1 - \mathrm{Q}_{\mu}\left(\sqrt{\frac{2\mu\delta_k t_1^{\alpha_1}}{1-\delta_k}}, \sqrt{\frac{2\mu}{1-\delta_k}} \left(\frac{\Omega\gamma_{\mathrm{th}}}{\bar{\gamma}}\right)^{\frac{\alpha_k}{2}}\right)\right] \mathrm{d}t_1. \end{aligned} \tag{4.2}$$

## 4.2 ERGODIC CHANNEL CAPACITY

Ergodic channel capacity is an important performance measure associated with reliable communication and is the theoretically maximum data rate that one communicate at over a fading channel.

The ergodic channel capacity $C$, for a FAS, can be derived as [2, Eq. (9)]

$$C = \int_0^{\infty} \left(\frac{1}{1+y}\right) \mathrm{Prob}\left(h > \sqrt{\frac{y}{\Theta}}\right) \mathrm{d}y. \tag{4.3}$$

Since $\mathrm{Prob}(h > a) = 1 - \mathrm{Prob}(h < a)$ and using (4.1), it follows that the ergodic channel capacity is written as

$$C = \int_0^\infty \left(\frac{1}{1+y}\right) \Bigg\{1 - \frac{\alpha_1 \mu^\mu}{\Gamma(\mu)} \int_0^{\sqrt{\frac{\Omega y}{\bar{\gamma}}}} t_1^{\alpha_1 \mu - 1} \exp\left(-\mu t_1^{\alpha_1}\right)$$

$$\times \prod_{k=2}^{N} \left[1 - \mathrm{Q}_\mu\left(\sqrt{\frac{2\mu\delta_k t_1^{\alpha_1}}{1-\delta_k}}, \sqrt{\frac{2\mu}{1-\delta_k}\left(\frac{\Omega y}{\bar{\gamma}}\right)^{\frac{\alpha_k}{2}}}\right)\right] \mathrm{d}t_1 \Bigg\} \mathrm{d}y \tag{4.4}$$

# 5 RESULTS

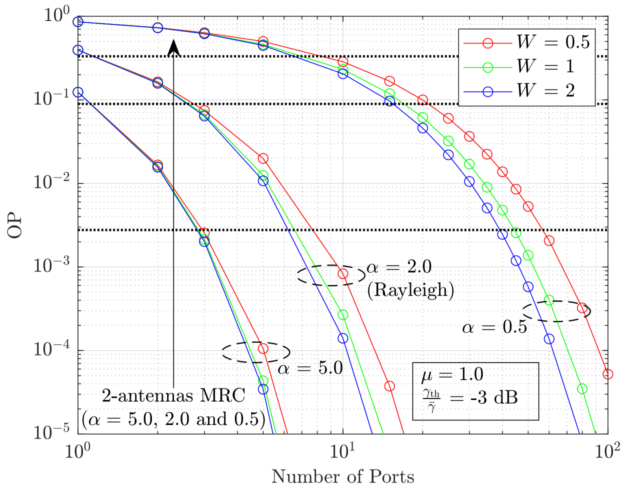
In this chapter, theoretical OP, channel capacity, and LCR curves are presented. Theoretical curves were carried out by using the MATLAB software according to the models described throughout the paper.

## 5.1 NUMERICAL RESULTS

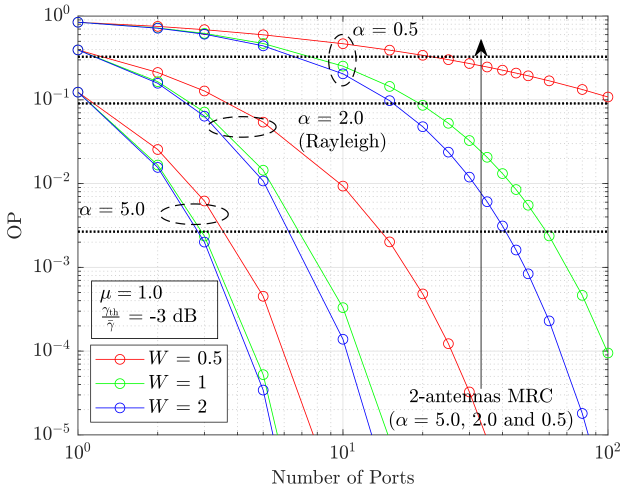
In Fig. 5.1, the OP as a function of the number of ports $N$ are presented. In our analysis, $\mu = 1.0$, $W = \{0.5, 1, 2\}$, $\alpha_1 = \alpha_2 = \cdots = \alpha_N = \alpha = \{0.5, 2, 5\}$ and $\gamma_{\text{th}}/\bar{\gamma} = -3$ dB. In Fig. 5.1, the influence of the non-linearity parameter $\alpha$ and the size $W$ in the OP is depicted. In Fig. 5.1(a), the power correlation coefficient $\delta_k$ presented in (2.1) is adopted. In Fig. 5.1(b), the OP curves are presented considering the new and improved model for $\delta_k$, as defined in (2.2). Considering [13, Eq. (5)] and using the same approach presented in the aforementioned article, it follows that

$$P_{\text{out}} = \int_0^\infty \frac{t_1^{\mu-1}}{\Gamma(\mu)} \exp\left(-t_1\right) \prod_{k=1}^{N} \left[1 - \mathrm{Q}_\mu\left(\sqrt{\frac{2\delta t_1}{1-\delta}}, \sqrt{\frac{2\mu}{1-\delta}} \left(\frac{\Omega \gamma_{\text{th}}}{\bar{\gamma}}\right)^{\frac{\alpha_k}{2}}\right)\right] \mathrm{d}t_1. \tag{5.1}$$

As the number of ports or $W$ increase, the OP value in Fig. 5.1 decreases for a fixed value of $\alpha$, since the power correlation coefficient decreases. It is possible to realize in Fig. 5.1 that for a fixed value of $N$ or $W$, as $\alpha$ increases, i.e. the less severe the fading is, the lower is the value of the OP, since that $\gamma_{\text{th}} < \bar{\gamma}$. As the higher the values of $\alpha$ are, the more deterministic the channel is around the $\bar{\gamma}$ value. Therefore, when $\gamma_{\text{th}} < \bar{\gamma}$ the probability of the SNR to fall bellow of $\gamma_{\text{th}}$ decreases. As a benchmark, OP curves under Rayleigh fading are shown. In fact, by making $\mu = 1.0$, $\alpha = 2.0$ in (4.1) or (5.1), the OP expressions presented in [3, Eq. (16)] or [13, Eq. (16)] can be easily obtained, respectively. For comparison purposes, the performance of the FAS is confronted to that using the maximal ratio combining (MRC) technique with 2 antennas, in which it is evidenced that, for a certain number of ports, the FAS has a better performance. In Fig. 5.1(b), it is verified that the use of (2.2) allows a more realistic analysis of the FAS, in which there is a worse performance in all scenarios even for large $N$. Also in this case, FAS also outperforms a system that uses 2-antenna MRC.




(a)




(b)

Figure 5.1: OP for the FAS as a function of $N$, with $\delta_k$ given by (a) (2.1) and (b) (2.2).

Ergodic channel capacity curves as a function of $N$ are plotted in Fig. 5.2 for variable $\alpha = \{1, 2, 5, 10\}$, fixed $\mu = 1.0$, $\bar{\gamma} = 10$ dB, and $W = 0.5$. For $\mu = 1.0$, the Weibull fading model can be obtained as a particular case of the study proposed in this work. To the best of the authors' knowledge, results concerning the performance of FASs under Weibull model have not been presented in the literature. Capacity curves under Rayleigh fading are also shown in Fig. 5.2 as a benchmark, in which some results of [2, Fig. 3(a)] are reproduced as special case. It should be noted that [2, Eq. (11)] can be obtained from (4.4) under $\mu = 1.0$ and $\alpha = 2.0$. From Fig. 5.2, it is noted that the capacity improves as the parameter $\alpha$ decreases.

The normalized LCR $L(\rho_{\text{th}})/f_{\text{D}}$ as function of $N$ is shown in Fig. 5.3, with $\mu = 2.0$, $\rho_{\text{th}} = 25$ dBm and $W = 0.2$, for different values of $\alpha = \{1, 1.5, 2\}$. In [4], the normalized LCR for Rayleigh channels is presented. The expression described in [4, Eq. (6)] can be also obtained from our work. As observed in Fig. 5.3, the normalized LCR decreases with the increase of the parameter $\alpha$ (i.e., the LCR improves as the channel non-linearity decreases). For comparison, the normalized LCR considering $\delta_k = 0$ are also

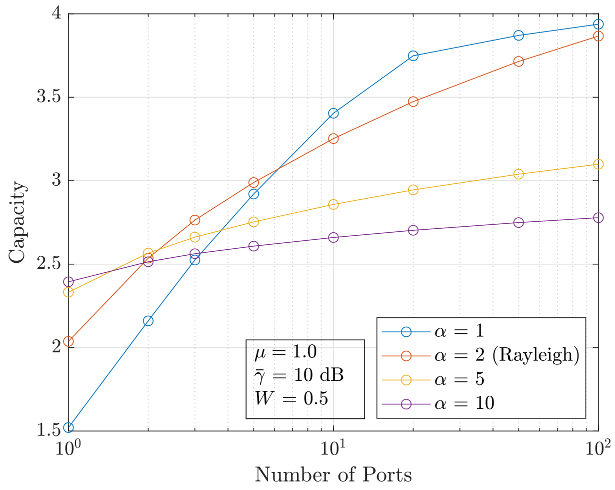



Figure 5.2: Ergodic channel capacity as a function of $N$.

presented, that corresponds to the case where the ports are independent. When $\delta_k = 0$, for all $\alpha$ values considered, it should be mentioned that the LCR is better. As shown in (3.17), it should be mentioned that the normalized LCR depends of the power correlation coefficient, the decision threshold and the channel and FAS parameters.

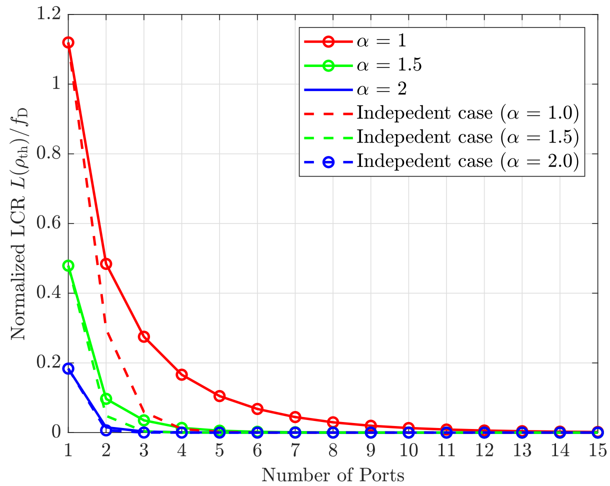



Figure 5.3: Normalized LCR $L(\rho_{\rm th})/f_{\rm D}$ as a function of $N$, under different $\alpha$ values, with $\mu = 2$, $\rho_{\rm th} = 25$ dBm and $W = 0.2$.

# 6 CONCLUSIONS

This work advanced the knowledge of FAS under $\alpha$-$\mu$ fading channels. In the mentioned context, important statistics were derived, such as the PDFs, CDF, and LCR. Metrics such as the OP and ergodic channel capacity were also deduced. Furthermore, an exact reduction of the OP due to $N$-th port for an $(N-1)$-port were also presented. Curves were shown for the OP and ergodic channel capacity as a function of the number of the ports under different parameters of fading model and the system, in which some insights were perceived. Furthermore, some results available in the literature were reproduced as particular cases of the study proposed.

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