Universidade de Brasília – UnB

Faculdade UnB Gama – FGA

Engenharia Aeroespacial

# Cubesat Sails: Use of laser beam to change cubesat orbit

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Brasília, DF

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Monografia submetida ao curso de gradu-
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Brasília, DF, :

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Brasília, DF

2021

*For future students*

# Acknowledgements

*"I am credited with being one of the hardest workers and perhaps I am, if thought is the equivalent of labor, for I have devoted to it almost all of my waking hours. But if work is interpreted to be a definite performance in a specified time according to a rigid rule, then I may be the worst of idlers. Every effort under compulsion demands a sacrifice of life-energy. I never paid such a price. On the contrary, I have thrived on my thoughts.".*

*(Nikola Tesla, Autobiography)*

# Abstract

A quick '*walk-through*' of the fundamentals is made to expose the similarities of the quantum mechanic and electromagnetic approach to reach the final equation describing radiation pressure. As radiation pressure is described by these two point of view, it is explicit that for a laser propulsion it is reasonable to work with electromagnetic approach as it ease the manipulation of the energy transfer by the high energy laser. The orbit dynamic of a satellite with solar sail have some particular differences in the equation of motion because of its behaviour receiving momentum transfer from a propulsion system external to its own body, and this require a series of force, coefficients and angle analysis. All these three parameters are exposed and calculated as well as the laser power efficiency that will reach the sail considering a simple ambient deviation of the beam, and these results are used to check the available ground sites. With all necessary paramaters calculated, the possible orbit change using 100kW laser power incidence in a sail with $32m^2$ and satellite weight $5kg$ results are positive and a good amount of change are achieved using this type of laser with the parameters given in this work. Until the date of this work a laser with 100kW power is not yet fully developed to achieve such power in the amount of time duration needed for this application, but a 60kW laser are being produced for military purpose and soon we will get a 100kW comercial laser. Theoretically it is feasable to use high energy laser to change cubesats orbits parameters.

**Key-Words**: Laser propulsion. solar sails. cubesat. lightsail.

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# List of symbols

| | |
|---|---|
| $E$ | Energy |
| $h$ | Planck's constant |
| $v$ | Frequency |
| $m_0$ | Mass of the body at rest |
| $c$ | Speed of light |
| $p$ | Momentum |
| $P$ | Pressure |
| $W$ | Energy flux at a distance $r$ from the Sun |
| $L_S$ | Solar luminosity |
| $W_E$ | Energy flux measured at Earth-Sun distance |
| $R_E$ | Sun-Earth distance |
| $A$ | Area |
| $t$ | Time |
| $f$ | Force exerted on the solar sail |
| $f_r$ | Force due to reflectance |
| $f_a$ | Force due to absorption |
| $f_e$ | Force due to emissivity |
| $f_n$ | Total force normal to the sail surface |
| $f_t$ | Total force tangent to the sail surface |
| $s$ | Specular reflection |
| $\alpha$ | Angle between the plane of the sail and the incident ray |
| $\varepsilon_f$ | Emissivity of sail's front |
| $\varepsilon_b$ | Emissivity of sail's back |
| $B_f$ | Lambertian coefficient of the sail's front |
| $B_b$ | Lambertian coefficient of the sail's back |

# Contents

# Introduction

The theoretical design of solar sails came before the idea to use laser beam as a photonic momentum transfer and it is agreed that Konstantin E. Tsiolkovsky were the first to suggest that a spacecraft could be propelled by sunlight in the publication of 1921 *Extension of Man into Outer Space*, but only in 1924 his student Fridrickh Tsander would publish the first paper about solar sailing (MCINNES, 2000). The solar sail propulsion system design had a lot of technical issues in it's initial phase such as the method of deployment and the material of the sail, but recently these issues were solved and missions to gather data for future researches were successfully accomplished. In 2010 Japan Aerospace Exploration Agency (JAXA)[1]. launched IKAROS[2], a satellite with built-in solar power that would be used as solar sail to have practical data about this type of propulsion. It was the first mission in the world to acquire this type of data along with information about solar power and sail deployment techniquies.

Studies about solar sail propulsion are getting attention as it is proving the feasibility in the use of sails for small satellites. The solar radiation momentum transfer are the topic of many studies about sails and the laser-driven propulsion still in the field of interplanetary projects, but here in this work I will try to cover a initial analysis of the laser propulsion, from earth to low earth orbit, being used for orbit change in cubesats.

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[1] <https://global.jaxa.jp//>
[2] <http://www.isas.jaxa.jp/en/missions/spacecraft/current/ikaros.html/>

# 1 Solar Sail Propulsion Fundamentals

## 1.1 Objetive

The main focus will be in the possibility of cubesats orbit change using laser-driven system and parameters of the LightSail 2 mission (SPENCER, 2020), but first introducing physics and mathematical fundamentals to achieve a reasonable result.

## 1.2 Radiation Pressure

Radiation pressure is a topic that was dicussed and studied a lot until any practical experiments were able to be made, due to this long theoretical time spent in studies of this phenomena some mathematical explanations were made, but only the latest two approaches to define radiation pressure is accepted today. By the quantum mechanics view, the radiation pressure happens because momentum is transported by photons (the quantum packets of energy that light is composed), and from the electromagnetic point of view the momentum is transported to the solar sail by electromagnetic waves (MCINNES, 1999). Even though the quantum mechanics approach will be described in this section, the electromagnetic model will be available in the laser-driven section. These two models give us exactly the same equation to work with.

### 1.2.1 Describing Radiation Pressure

A quick *'walk-through'* on the mathematical development to determine the pressure exerted on a surface due to sun radiation (MCINNES, 1999). For symbol clarification check the list of symbols.

Consider the special relativity mass-energy equivalence for a moving body;

$$E^2 = m_0^2 c^4 + p^2 c^2 \tag{1.1}$$

Where the first term on the right side of the equation is the rest energy of the body and the second is the motion energy.

Since a photon has zero rest mass the equation 1.1 becomes;

$$E = pc \tag{1.2}$$

To determine the momentum transported by a single photon we use Planck's law:

$$E = hv \tag{1.3}$$

And substituting equation 1.3 on equation 1.2;

$$p = \frac{hv}{c} \tag{1.4}$$

The energy flux $W_E$ considering the distance from Earth to the Sun is;

$$W_E = \frac{L_S}{4\pi R_E^2} \tag{1.5}$$

Then we can write the energy flux $W$ at a distance r from the Sun

$$W = W_E (\frac{R_E}{r})^2 \tag{1.6}$$

Using the energy flux we can obtain the energy $\triangle E$ transported in a surface area in time $\triangle t$

$$\triangle E = WA\triangle t \tag{1.7}$$

The equation 1.2, when solving for $p$ we see that the energy $\triangle E$ transports a momentum $\triangle p$

$$\triangle p = \frac{\triangle E}{c} \tag{1.8}$$

Now that we have described all the terms we need, it's time to write the pressure exerted on a surface area in terms of the momentum transported per unit time and area; one

$$P = \frac{1}{A}\frac{\triangle p}{\triangle t} \tag{1.9}$$

$$P = \frac{W}{c} \tag{1.10}$$

## 1.2.2 Describing Laser-driven propulsion

For the laser propulsion it is necessary to look at the energy transfer by the electromagnetic point of view. The complete derivation and explanation of this model is well documented in the book Solar Sailing, Technology, Dynamics and Mission Applications by Colin R. McInnes[1], thus making the whole presentation here unnecessary.

A electric field is considered and a component of it's wave induce a current $j$ in the sail. A Lorentz force $\mathbf{j}x\mathbf{B}$ is generated as a magnetic component of incident wave $\mathbf{B}$ appears in the direction of the propagation, see fig.1, and for this wave propagating along the x axis the force exerted is;

$$df = j_z B_y dx dy dz \tag{1.11}$$
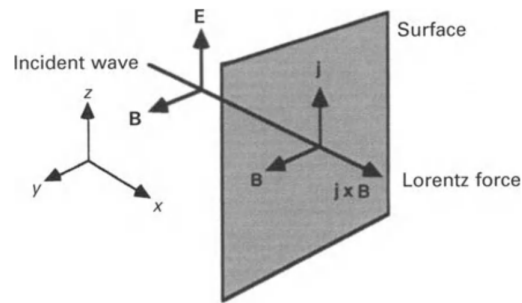


Figure 1 – Electromagnetic description of radiation pressure (MCINNES, 1999)

The pressure is defined to be force per unit area, thus the element in fig.1 will perceive a pressure written as;

$$dP = j_z B_y dx \tag{1.12}$$

Using Maxwell's equation we can get the field terms,

$$dP = \frac{\partial}{\partial x}(\frac{1}{2}\varepsilon_0 E_z^2 + \frac{1}{2\mu_0}B^2)dx \tag{1.13}$$

$$U = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0}B_y^2 \tag{1.14}$$

U being the energy density, E the electric component, B the magnetic component of the incident wave, $\varepsilon_0$ the permittivity of free space and $\mu_0$ the permeability.

---

[1]  (MCINNES, 1999)

$\triangle$l being the surface thickness, the pressure is obtained by integrating equation 1.16;

$$P = -\int_0^{\triangle l} \frac{\partial U}{\partial x} dx \tag{1.15}$$

The pressure exerted will be the total energy density of the electromagnetic wave $P = U$.

For the case shown in fig. 2 we have a distance $\triangle x$ between those two plane waves with incident occurring on a surface area $A$.
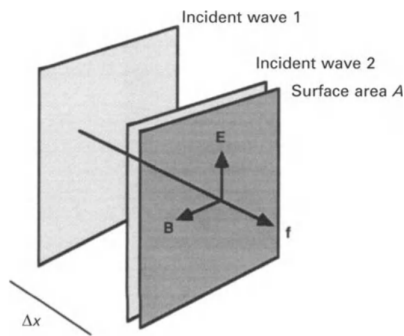


Figure 2 – Energy density of an electromagnetic wave (MCINNES, 1999)

The energy density of the electromagnetic wave, considering that $\triangle x$ is equivalent to $c\triangle t$, we have;

$$U = \frac{\triangle E}{A(c\triangle t)} \tag{1.16}$$

Knowing that the energy flux $W$ across the surface is;

$$W = \frac{\triangle E}{A\triangle t} \tag{1.17}$$

we reach a pressure equation based on the energy density of the electromagnetic wave;

$$U = \frac{W}{c} \tag{1.18}$$

And as expected it is equivalent to the equation 1.10.

# 2 Orbit Dynamic for Solar Sail

## 2.1 Equation of motion

This section will not cover the sun-centered orbit. We are interested in the the Earth-Centered system to take a look at the feasibility of the laser beam system.

The analysis of the orbit will include a look at some controls of the sail to keep desireable aspects of its force vectors. As it will receive a beam light from the Earth it is needed to keep an angle to receive the photon transfer properly and manage these vectors in the right direction. Considering more than one ground site to be the source of the laser beam a control over the sail angle with respect to earth will be needed.

According to the general equation of motion for a perfectly reflecting solar sail (MCINNES, 1999);

$$\frac{d^2\mathbf{r}}{dt^2} + \mu \frac{\mathbf{r}}{r^3} = \kappa(\mathbf{l} \cdot \mathbf{r})^2 \mathbf{n} \tag{2.1}$$

And in this adaptation the vector $l$ in the equation above is considered the unit vector directed along the laser beam line, $\kappa$ being the magnitude of the solar sail acceleration (A metric parameter to classify performance of a solar sail based on the mass, area and efficiency it has) (MCINNES, 1999).

We are really interested in the use of the normal and transverse force, eq.2.3 and eq.2.4. As we might expect there will be points of the orbit where the ground site will not be able to deliver the beam as intended to use the maximum approximation of normal force, and in that case we have the transverse force to work with.
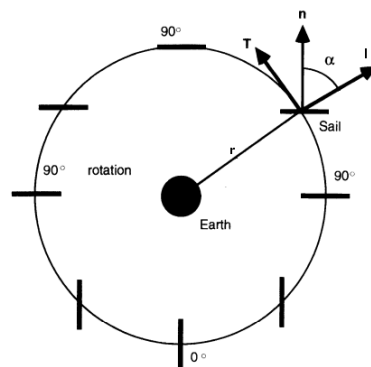


Figure 3 – Force vectors illustration of a solar sail with Earth perspective (MCINNES, 1999) Adapted

## 2.2   Forces and Coefficients

The Jet Propulsion Lab of California Institute of Technology[1] in 1978 made an optical force model for solar sail during the project to make a mission to Halley's Comet. Since that time this model became a standard, because it included tests to determine optical coefficients using a thickness of 2.5 micrometer Kapton sail material coated with 100 nanometer of aluminium on the front side and chromium on the back side (HEATON; ARTUSIO-GLIMPSE, 2015). This model take into account reflectance, absorption and emissivity of the sail film and its total force is represented as;

$$f = f_r + f_a + f_e \tag{2.2}$$

According to these contributions and the derivation in the literature we reach to these two equations [(MCINNES, 1999), p.47],(GRESCHIK, 2013);

$$f_n = PA[(1 + \tilde{r}s)\cos^2\alpha + B_f(1 - s)\tilde{r}\cos\alpha + (1 - \tilde{r})\frac{\varepsilon_f B_f - \varepsilon_b B_b}{\varepsilon_f + \varepsilon b}\cos\alpha] \tag{2.3}$$

$$f_t = PA(1 - \tilde{r}s)\cos\alpha\sin\alpha \tag{2.4}$$
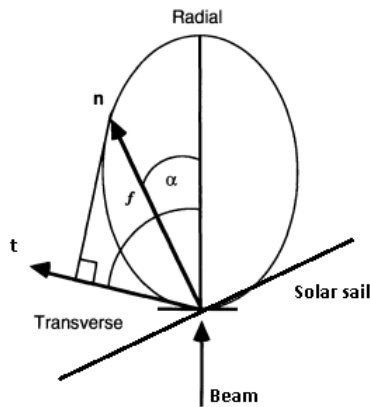


Figure 4 – Force vectors of a solar sail (MCINNES, 1999) Adapted

As $\varepsilon$ being the emissivity and $B$ the Lambertian[2] coefficient, both identified by $f$ for front side of the sail and $b$ for the back. The $\tilde{r}$ is reflection coefficient and $s$ the specular reflection. These coefficients can be found in an updated article that derived them again to use in new missions (HEATON; ARTUSIO-GLIMPSE, 2015). In this work the updated values will be used as seen in the table below ;

---

| Coefficient | $\tilde{r}$ | $s$ | $B_f$ | $B_b$ | $\varepsilon_f$ | $\varepsilon_b$ |
|---|---|---|---|---|---|---|
| Value | 0.91 $+/-0.005$ | 0.94 $+/-0.04$ | 0.79 $+/-0.05$ | 0.67 $+/-0.05$ | 0.025 $+/-0.005$ | 0.27 $+/-0.005$ |

Table 1 – Coefficients value

Before we leave this section it is important to calculate the irradiance of the laser to start our analysis. The table 2 bellow holds all the useful information to use the irradiance code made to calculate the ideal irradiance (without considering jitter, that is, not considering the deviation of the laser pulse) and the approximation irradiance including jitter. The code used was based on studies made by Paul H. Merritt and John R. Albertine (MERRITT; ALBERTINE, 2012)

| Laser Parameters | |
|---|---|
| Power | 100 [kW] |
| Laser-Aperture | 0.3 [m] |
| Object Distance | 1015 - 1215 [km] |
| Wavelength | 1024 [nm] |
| Jitter | 3 [$\mu$ rad] |

Table 2 – Laser parameters based on Eglin Air Force Base ground site used as reference for a 10kw laser (modified to 100kw here) (IV; THOMAS, 2016)

Results in ideal case and with jitter considered are in table 3 bellow;

| Irradiance | |
|---|---|
| Ideal | 6543.3536 $[W/m^2]$ |
| With jitter | 1427.2795 $[W/m^2]$ |

Table 3 – Irradiance calculated for a laser with parameters in table 2

## 2.2.1 Forces exerted on Sail with 100kW laser beam

The ideal irradiance mentioned in previous section and shown in table 3 is the peak irradiance using the laser, due to no jitter considered it is the maximum irradiance possible for the range and parameters considered in table 2. But this high value is almost surreal to acquire. Using equation 1.18 to get the radiation pressure magnitude,with jitter, we have;

$$U = \frac{792.204}{299792458} = 4.7608.10^{-6} N/m^2 \tag{2.5}$$

As stated in section 1.2.2 that $U = P$ we now look foward to calculate the normal force generated from equantion 2.3. Considering the coeficients in table 1 and using the code in appendix A.2 we have that the normal force will be in the magnitude of $1.9860.10^{-4}N$. The variation of $\alpha$ (Angle between the plane of the sail and the incident beam) versus the normal force generated is shown in figure
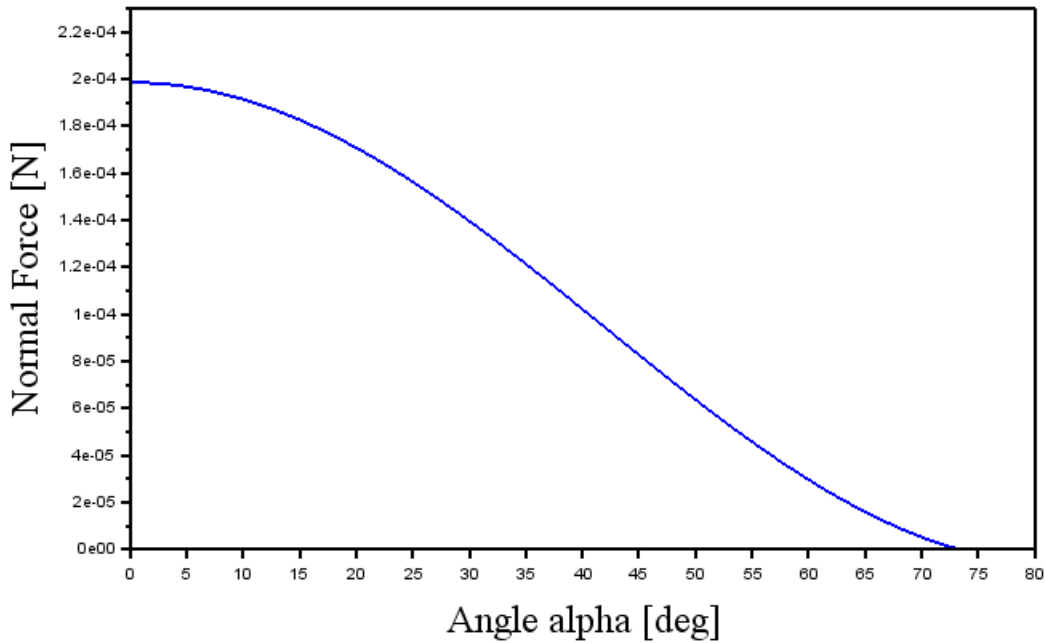
Figure 5 – Normal force magnitude [N] versus Angle $\alpha[°]$

We see that the normal force becomes irrelevant when $\alpha$ approaches 75° and as expected the maximum occurs at 0°, that is, when the incident laser beam line is coincident with the normal vector of the sail. Now we will take a look at the transverse force using the equation 2.4 and implemented in the code at appendix A.3;
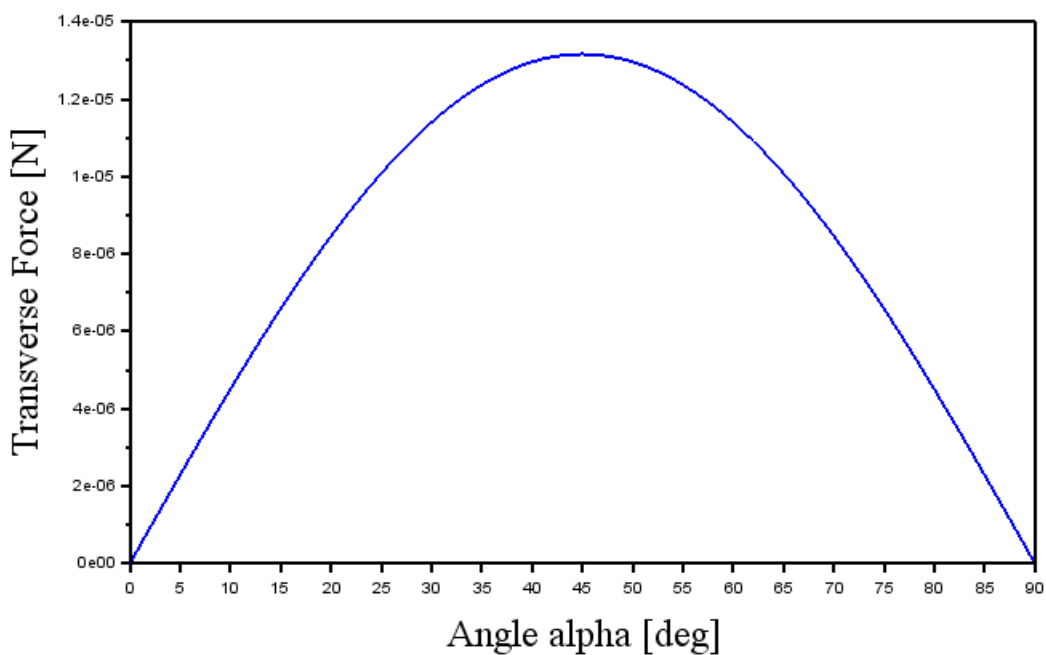


Figure 6 – Transverse force magnitude [N] versus Angle $\alpha[°]$

The maximum in transverse force occurs at approximately 45° with a magnitude of $1.32.10^{-5}N$. This shows that in order to get maximum transverse force the normal vector force must be at 45° with respect to the incident laser beam line reaching the sail. These were the magnitude that forces components can achieve with such laser parameters, but we must know the direction of resultant force vector. (MCINNES, 1999)
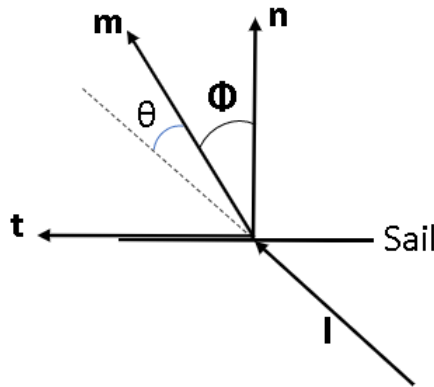


Figure 7 – Angle $\phi$ and $\theta$ visualized

In figure 7 angle $\theta$ is the one between incident beam **l** and total vector force **m**, this is the cone angle (MANCHESTER; LOEB, 2017). For $\phi$ we see it is the angle between normal force **n** and total vector **m** called centre-line angle. These two angles can be used to know the direction of the total vector. For $\phi$, it can be calculated by simply getting the tangent of $\phi$ as follows; (MCINNES, 1999)

$$tan\phi = \frac{ft}{fn} \qquad (2.6)$$

This centre-line angle is important to know the angle constraint of our sail. The figure 8 shows the relation of $\phi$ with $\alpha$. With $\phi = 0$ meaning that the total force vector **m** is at the normal direction of the sail, thus $\alpha = 0$ because in order to have **m** completely normal to the sail we need no angle between the incident beam and the normal vector. As $\alpha$ increase (the sail change its angle relative to the incident beam) the force vector **m** will change its angle $\phi$ slowly until we reach the maximum angle the sail can achieve to have a positive force vector as a result from $f_n$ and $f_t$.
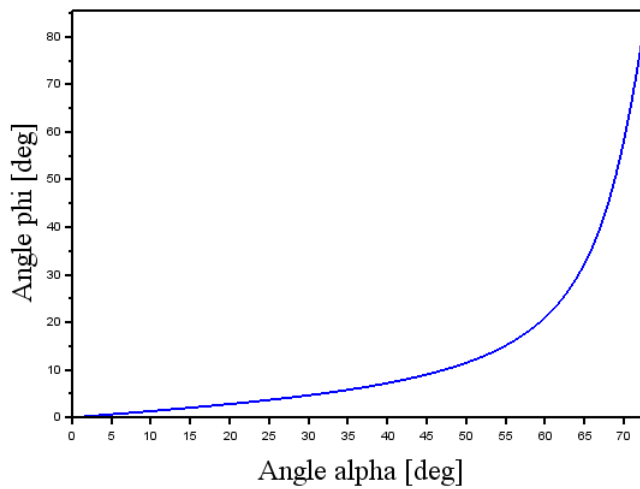
Figure 8 – Angle $\phi[°]$ versus Angle $\alpha[°]$

For a better perspective of what type of contraint the figure 8 brings to us, lets take a look at the figures bellow showing how $\phi$ and $\alpha$ are related;



(a) Ideal situation $\alpha = 0°$             (b) $\alpha = 30°$

(c) $\alpha = 55°$                           (d) $\alpha = 74°$
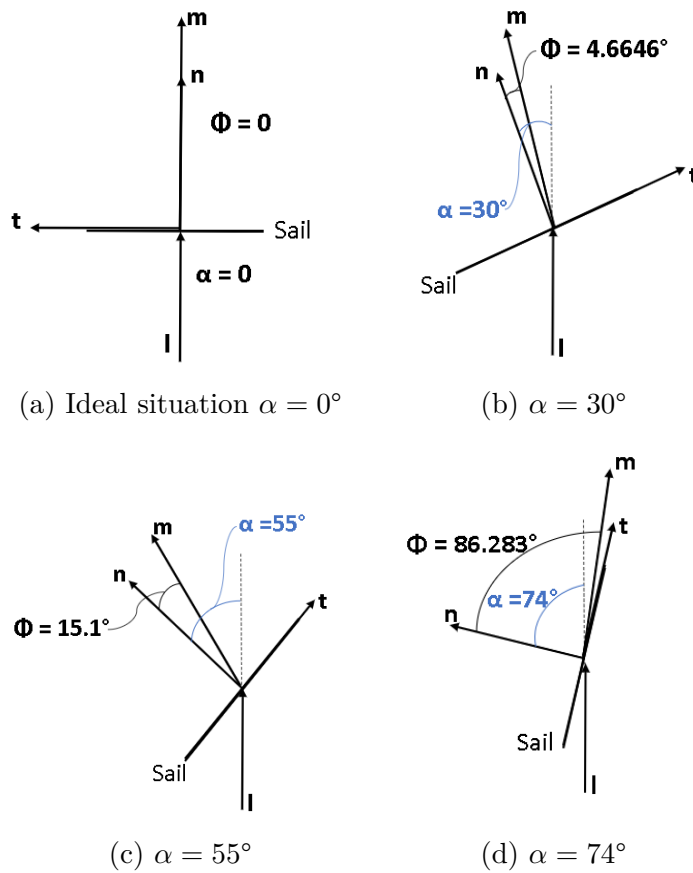
Figure 9 – Visualization of $\alpha$ influence on $\phi$

## 2.3   Ground Site

Elegible ground sites that are already built and have safety requirements were analyzed based on the latitude, longitude and altitude (IV; THOMAS, 2016). A few of them are shown in figure 10;

| Ground Site | Latitude (deg) | Longitude (deg) | Altitude (km) |
|---|---|---|---|
| Haleakala | 20.7085 | -156.258 | 3.057 |
| Huntsville, AL | 34.6064 | -86.6557 | 0.171 |
| Kwajalein | 8.71955 | 167.719 | 0.05904 |
| North Obscura Peak, NM | 33.7522 | -106.372 | 2.400 |
| Santa Cruz | 37.1399 | -122.202 | 0.710 |
| Santa Rosa Island, FL | 30.3979 | -86.7291 | 0.000 |
| Starfire Optical Range | 34.9642 | -104.464 | 1.871 |
| White Sands | 32.6325 | -106.332 | 1.205 |

Figure 10 – Ground sites elegible to host the laser system (IV; THOMAS, 2016)

Studies made by NASA (National Aeronautics and Space Administration) about the LightSail 2 mission shows that for a ground site located in Santa Rosa Island the dwell time for laser opportunities is 397 seconds (24° inclination circular orbit at 700km altitude), divided in two access times of orbit pass (IV; THOMAS, 2016). Remembering that depending on the laser power and location of the system on the Earth surface we have to consider the angle constraints mentioned in figure 9. Here the analysis of a proper ground site will be as simple as possible due to the lack of information granted by these governamental organization about the building itself.

As mentioned, the angle that a satellite sees the laser ground site have some constraints. This angle is called *nadir* and is represented in the figures bellow;
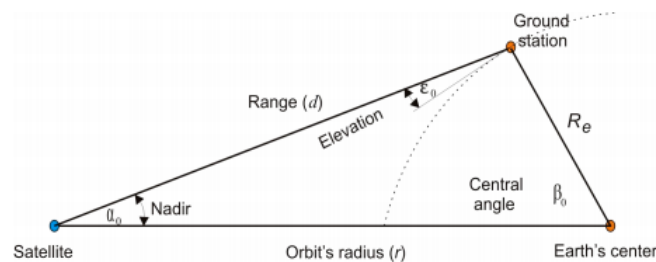


Figure 11 – Ground station geometry 1 (CAKAJ, 2011)
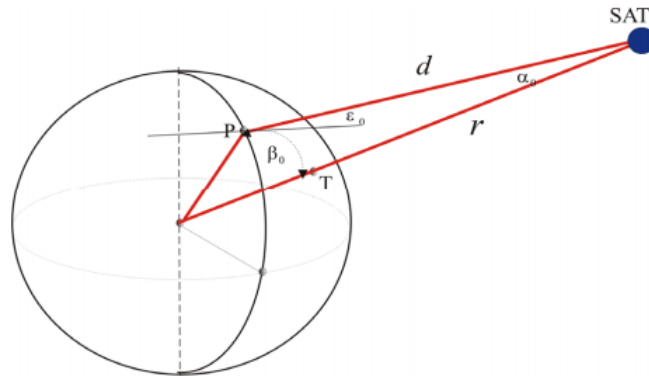
Figure 12 – Ground station geometry 2 (CAKAJ, 2011)

With the method presented by Cakaj (CAKAJ, 2011) a slant range can be calculated for a generic orbital altitude considering the elevation angle of the ground site as shown in figure 13;
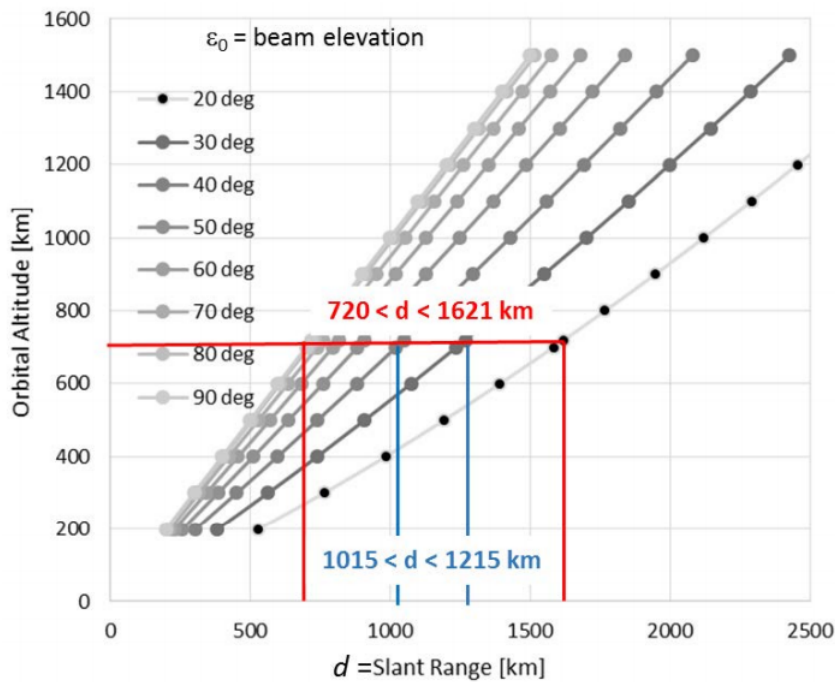


Figure 13 – Laser propagation distance from the ground to spacecraft (IV; THOMAS, 2016)

In figure 13 the red line is highlighting the slant range of the LightSail 2 mission, that is; 720km altitude orbit. This analysis shows that opportunities of laser access will not have the same slant range, therefore, the power delivered to the sail by the ground site will change inside the dwell time. The blue line indicated in the figure 13 is the optimal range inside the nadir angle of 30° and 40°. These limit angles starting on 30° are the results of studies made by radio amateur (BRUNINGA, ) about circular orbits and its dwell time with respect to the elevation angle (nadir), and concluding that only 40% of the dwell time is elegible to receive the laser beam.

# 3  Amount of change in orbit

Now that we have considered and explained the parameters needed, the $\triangle V$ possible will be calculated using orbit parameters from LightSail 2 mission. Since cubesats have a relative similar weight, size, and ballistic coefficient this study results might include other missions to the possibility of laser beam use in LEO.

The orbit parameters that will be considered in this section are shown in the two-line element set (TEL) bellow;

```
1 44420U 19036AC   21026.42173555  .00008150  00000-0  16776-2 0  9999
2 44420  24.0066 136.3264 0019102  46.4956 313.7122 14.58300401 82821
```

Figure 14 – Two-line element set from LightSail 2 mission

These parameters were taken in 26 January of 2021 at 10:07:17.[1] For better view and identification, a table with parameters that we need are in the table bellow;

| | |
|---|---|
| Eccentricity | 0.0019102 |
| Inclination | 24.0066° |
| Perigee height | 685 [km] |
| Apogee height | 712 [km] |
| Right ascension of ascending node | 136.3264° |
| Argument of perigee | 46.4956° |
| Revolutions per day | 14.58300401 |
| Mean anomaly at epoch | 313.7122° |
| Orbit number at epoch | 8282 |
| Mass | 5 [kg] |
| Sail Area | 32 $[m^2]$ |

Table 4 – LightSail 2 orbit elements

With the elements we are able to calculate the state vector, position and velocity vectors in 3D, and apply the possible velocity change generated by the laser beam. First, we need to know the magnitude of forces that will be applied in the situation of the satellite. Using the values acquired in section 2.2 for the normal and transverse force, and ground site dwell time parameters (with slant range) we can have the amount of acceleration delivered to the sail by dividing the force(thrust) by the mass of the satellite.

$$ac_n = \frac{f_n}{5[kg]} \tag{3.1}$$

---

[1]  <https://www.heavens-above.com/orbit.aspx?satid=44420>

$$ac_t = \frac{f_t}{5[kg]} \tag{3.2}$$

These acceleration are shown together in figure 15 bellow, so we can notice that it is balanced out as the magnitude of them are the same when the $\phi = 45°$. As previously demonstrated $\phi$ is the direction angle of the resultant force with respect to the normal vector. Since the normal vector is always perpendicular to the transverse vector, when $\phi = 45°$ the resultant vector is in the middle of both normal and transverse demonstrating the equilibrium of the calculation made through the work.
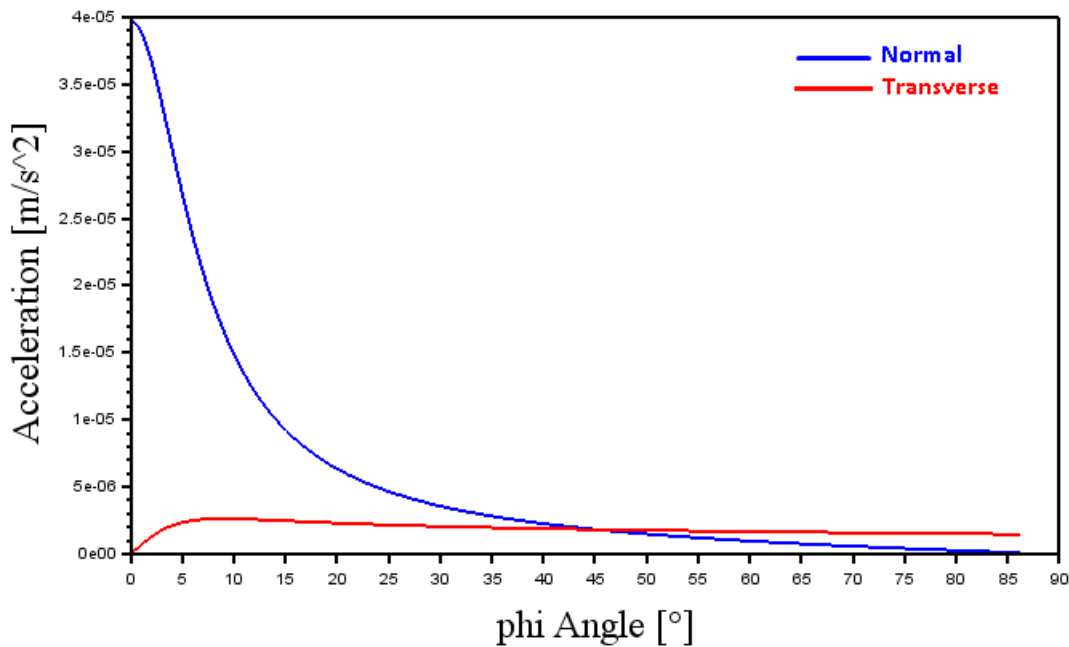


Figure 15 – Normal and transverse acceleration due to $\phi$ angle

The maximum normal acceleration happens when $\phi = 0°$, therefore, being at full transference of the incident beam. But the transverse acceleration deserves a better look to expose its maximum magnitude, as a mistake could be made thinking that the more higher $\phi$ get, more would be the magnitude of the transverse vector. This is not the case, as $\phi$ depends on the angle of beam incidence $\alpha$.

The transverse acceleration maximum occurs at approximately $\phi = 9°$ (figure 16) then starts decreasing until the last viable $\phi$ angle. It shows that in order to have maximum transverse acceleration the normal must be considered, because at this angle the normal still have expressive values.
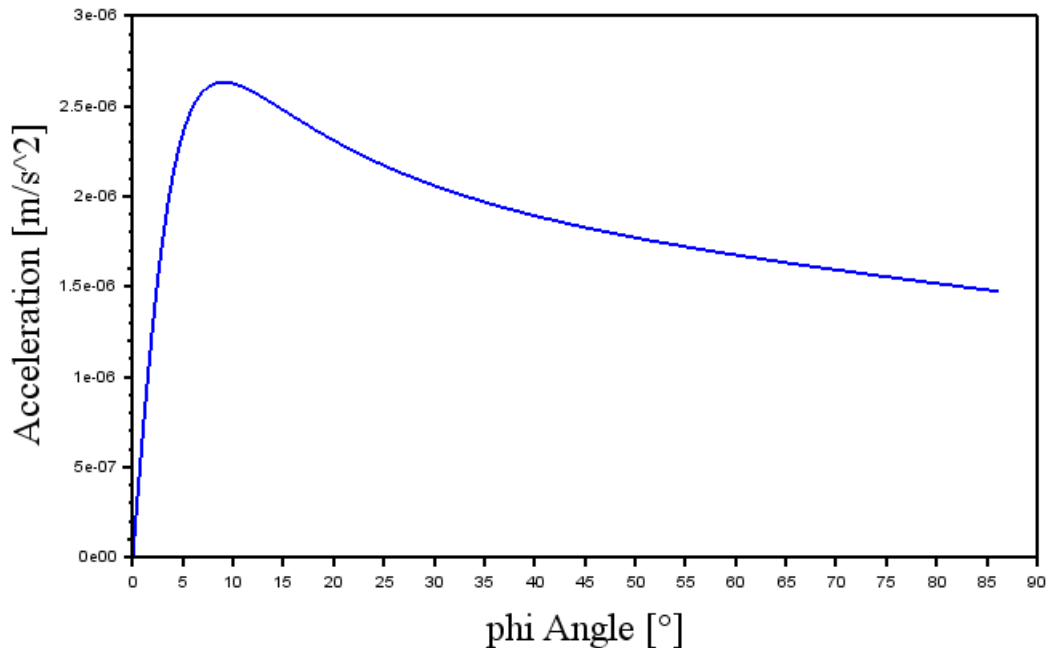
Figure 16 – Transverse acceleration due to $\phi$ angle

Now we can check the amount of acceleration transfer that will occur in a opportunity similar to the one described in section 2.3, where the dwell time disponible for the ground site located in Santa Rosa Island is 397 seconds. Considering the slant range and the optimal nadir angle mentioned in that section, the viable time is about 40% of the total satellite appearance leading us to a dwell time of 160seconds. For both, normal and transverse forces we calculate the velocity change by;

$$\triangle V_i = a_i.160 \tag{3.3}$$

Change in velocity vectors will always depend on the objetive of the maneuver, but to use as an example of how the laser-driven propulsion would affect an orbit we can make a few hypotetical assumptions.

Let $\phi = 9°$ be our fixed angle through the entire laser incidence, therefore, $\alpha = 45°$. The table bellow have all the parameters assuming this initial condition;

| $\phi$ | 9° |
|---|---|
| $\alpha$ | 45° |
| Normal Acceleration | $1.6558.10^{-5}\ [m/s^2]$ |
| Transverse Acceleration | $2.6325.10^{-6}\ [m/s^2]$ |
| Dwell time | 160 [seconds] |
| $\triangle V_{NInitial}$ | $2.6493.10^{-6}$ [km/s] |
| $\triangle V_{NFinal}$ | $1.8489.10^{-6}$ [km/s] |
| $\triangle V_{TInitial}$ | $4.2120.10^{-7}$ [km/s] |
| $\triangle V_{TFinal}$ | $2.9396.10^{-7}$ [km/s] |

Table 5 – Parameters for $\phi = 9°$

The amount of $\triangle V$ is different from the initial moment to the final simply because of the slant range, that starts at 1015km up to 1215km, meaning the power delivered by the laser beam is less effective. Now we can take a look at the satellite mentioned in table 4 and to know its position and velocity in a 3D vectorization orbit. A Scilab code were made based on the algorithm disponible in the book Orbital Mechanics for Engineering Students (MCINNES, 1999). Bellow are these vectors forms and values calculated;

$$\mathbf{r} = \begin{bmatrix} X & Y & Z \end{bmatrix}$$

$$\mathbf{r} = \begin{bmatrix} -5128.9029 & 4864.3875 & 10.430225 \end{bmatrix}$$
$$\mathbf{v} = \begin{bmatrix} -4.7128079 & -4.9906925 & 3.057051 \end{bmatrix}$$

For coordinate reference of this orbit the figure 17 shows the unit vectors of transverse, radial, and normal directions. The normal direction $\hat{\mathbf{h}}$ in the figure 17 is with respect to the plane of orbit, thus different from the normal direction metioned before in this work. And to finish the clarification of vectors directions, the radial vector $\hat{u}_r$ is the normal vector of the sail and the perpendicular vector $\hat{u}_{\perp}$ is our transverse vector.
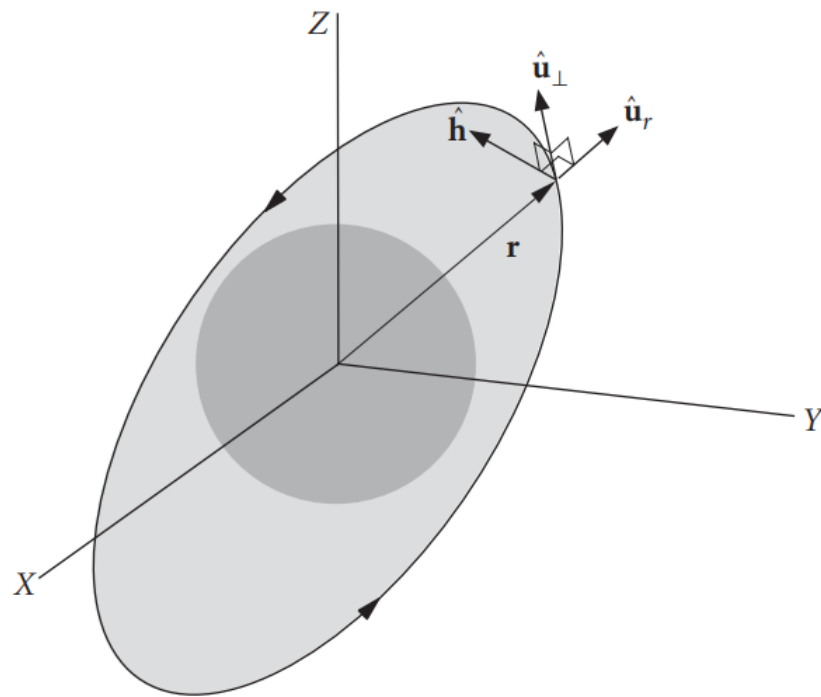
Figure 17 – Coordinates illustration (CURTIS, 2005)

Before applying the velocity changes it is necessary to adjust the coordinate system from the geocentric equatorial frame to the perifocal frame, then we will be able to add the velocity change to the vectors. After this rotation of matrix and increment in the velocity vector we must do the inverse path to have our state vectors in the geocentric equatorial frame again. For better understanding, the figure bellow shows a visualization of these two frames and its axes;
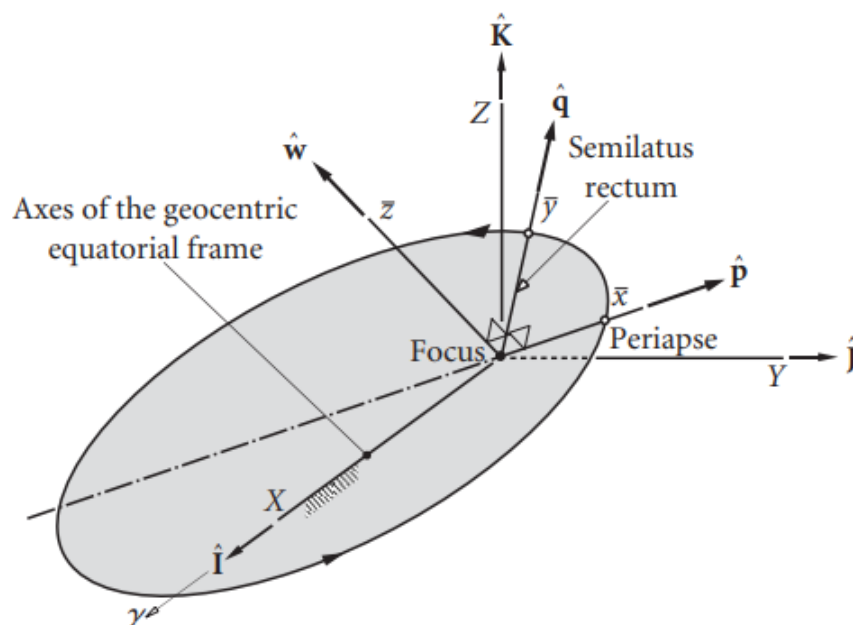


Figure 18 – Geocentric equatorial frame and Perifocal frame illustration (CURTIS, 2005)

The rotation matrix to find the perifocal terms is in the form bellow(CURTIS, 2005);

$$\mathbf{Q} = \begin{bmatrix} (\cos\Omega\cos\omega - \sin\Omega\sin\omega\cos i) & (\sin\Omega\cos\omega + \cos\Omega\cos i\sin\omega) & (\sin i\sin\omega) \\ (-\cos\Omega\sin\omega - \sin\Omega\cos i\cos\omega) & (-\sin\Omega\sin\omega + \cos\Omega\cos i\cos\omega) & (\sin i\cos\omega) \\ (\sin\Omega\sin i) & (-\cos\Omega\sin i) & (\cos i) \end{bmatrix}$$

Now we have the position and velocity vector in the perifocal frame, allowing us to add the $\triangle V$ we have;

$$r_p = \begin{Bmatrix} -1.5182.10^3 \\ -6.8953.10^3 \\ 0 \end{Bmatrix} [km]$$

$$v_p = \begin{Bmatrix} 7.3431 + \triangle V_T \\ -1.5993 + \triangle V_N \\ 0 \end{Bmatrix} [km/s]$$

Applying the velocity changes of table 5 to the velocity vectors previously mentioned will, in theory, cause the orbit to change if there is significant amount of acceleration transfer.

Now we retrieve the state vector to check the change in the orbital elements. It is important to remember that this type of system is design to give significant changes by multiple incidence times for months. Here we will be adding the value equivalent to 365 days of beam incidence two times a day for 160 seconds each dwell time. To check the orbital parameters of the satellite after this change we will be using another Scilab code made based on the algorithm on the book Orbital Mechanics for Engineering Students (CURTIS, 2005).

$$\mathbf{r} = \begin{bmatrix} 178.01635 & -5648.8545 & -4227.0431 \end{bmatrix}$$
$$\mathbf{v} = \begin{bmatrix} 3.9277737 & 3.9326405 & -5.0604931 \end{bmatrix}$$

| Elements | Before | After |
|---|---|---|
| Period | 5900.5092 | 5896.9219[seconds] |
| Velocity magnitude | **7.5152**[km/s] | **7.5167**[km/s] |
| Mass | 5 [kg] | 5 [kg] |
| Sail Area | 32 $[m^2]$ | 32 $[m^2]$ |

Table 6 – LightSail 2 orbit elements after laser beam incidence

The velocity change were in the magnitude of 0.0015237[$km/s$]. All this analytical approach might have its inherent errors and there are some characteristcs not considered here as the solar flux activity and aerodynamics aspects, because the goal is to check the amount of influence the laser alone can have in this case. A better visualization of it would be to take this analysis to simulation softwares such as System Tools Kit (STK) to check all the parameters and behaviour.

## 3.1 Drag enhancement

The magnitude of the total velocity change is low when compared with other options we already have to change orbits. This is the case where the technology should be used to enhance another one to achieve the final objective, and in this case the objective is to increase the drag of the satellite to deorbit it. In this section the drag force due to the influence of the laser beam will be shown, and the density model used was MSISE-90 Model of Earth's Upper Atmosphere.

The drag force is a well known force and the difficulties to calculate it correctly are the uncertainty associated with drag coefficient and density, but for theoretical drag coefficient the value for a sphere is 2 and for a flat plate is 4 (GUGLIELMO SANNY OMAR; JOHNSON, 2018). Here we will be using a fixed value of 2.2 for the drag coefficient, as some articles shows that for some cubesats missions this is the value used in the simulations before the launch(COTTEN; ZEE, 2017). To calculate the drag force properly the relative velocity of the satellite must be acquired:

$$\mathbf{v}_\infty = \mathbf{v} - \boldsymbol{\omega}\mathbf{x}\mathbf{r} \tag{3.4}$$

As $v_\infty$ is the relative velocity of the satellite considering the rotation rate of the Earth, $\mathbf{v}$ is the orbital velocity vector and $\mathbf{r}$ the position vector. This relative velocity is needed to calculate the drag force on the sail using a well known equation;

$$\mathbf{F}_d = -\frac{1}{2}C_d\rho A\mathbf{v}_\infty^2 \tag{3.5}$$

$F_d$ is the drag force in vector form acting on the sail by the influence of the particles hitting it, generating more drag we can achieve faster reentry plans, or make

adjustments in the orbit using it. Using the data already mentioned in this work the drag
force influenced by the laser beam is calculated and shown in the table 7 ;

|   | 1 | 2 | ... | 730 | 731 |
|---|---|---|-----|-----|-----|
| **X** | -3.056656**213299972**E-5 | -3.0566562**134510854**E-5 | ... | -3.056656**305349128**E-5 | -3.0566563**054545824**E-5 |
| **Y** | -3.427770**783278481**E-5 | -3.42777078343205E-5 | ... | -3.4277708**768235865**E-5 | -3.427770**876930755**E-5 |
| **Z** | -1.2862482263104288E-5 | -1.2862482263104288E-5 | ... | -1.2862482263104288E-5 | -1.2862482263104288E-5 |

Table 7 – Magnitude of drag force $F_d$ applied by laser beam each time the satellite passes
         by

The table shows 731 iterations not because it was applied 731 days, it is 365 days
2 times per day. It is an odd looking table, but shows how little the drag force (specifically
applied by the beam and density) rise as the satellite decay more and more. The rise is in
the order of $9.215.10^{-13}$N for X and $9.365.10^{-13}$N for Y from day 1 to day 365, and for Z
we won't have any change because the assumption before was an ideal case that the beam
would only affect X and Y in the perifocal frame, where Z is assumed to be zero in the
plane. Now it is time to bring the magnitude of the resultant drag force vector by taking
its norm and organizing it in 365 days chart with the cumulative change calculated. It was
expected to see a exponential rise in the cumulative contribution in the drag force, but as
table 7 shows we wouldn't notice any exponential behaviour with that type of magnitude
rise, only a linear growth in drag force.

## 3.2   Quick look at orbit lifetime changes

The orbit lifetime depends on the scale height value and according to Kallmann-
Bijl's scale height model, the average value of the scale height for altitude of 700km is
90.5km (KALLMANN-BIJL, 1961). The attempt to calculate the orbit lifetime in this
case was made using Desmond King-Hele work written in the book *Theory of satellite
orbits in an atmosphere* (KING-HELE, 1964), and it goes in terms of the period and its
rate of change. Clearly there are other methods and ways to have better accuracy, but
this section is just a quick look at the orbit lifetime in the idealization of the laser beam
influence in the lightsail2 cubesat.

The orbit lifetime is given by(KING-HELE, 1964);

$$t_L = \frac{3HT_c n}{2a_c \dot{T}_c} \tag{3.6}$$

Where H is the scale height, $T_c$ is the period in a circular orbit, $a$ is the distance
from the center of the Earth, $\dot{T}_c$ is the period rate of change, and $n$ is equal to 1 in circular
orbits but if accuracy is really needed it is better to calculate it (KING-HELE, 1964).
The first data acquired of the lightsail2 orbit was 26 January of 2021 at 10:07:17 and its
period were 5924.70522 seconds, and following the data to 270 days later 23 October of

2021 at 03:51:57 the period were 5912.52514. Using this data it is possible to produce $\dot{T}$ and expand linearly the amount of period change in this time, and we have $\dot{T}$=-0.00075186 min/day. This lead us to an orbit lifetime in the beginning of 2515.8 days (6.89years) and at the end of the 270 days the result is 2514.3 days (6.88years) for the laser beam alone, not considering the natural decay of the satellite. If we consider the natural decay using this method, we have a difference of 5.7218 days from the begining to the end, that is almost 5 times greater than the influence of the beam.

## 3.3   Quick look at internal efficiency

The conventional parameters to check efficiency in rocketry is not suitable for this case as it rely on specific impulse, and the definition is; specific impulse is equal to the momentum gained by the rocket per unit weight of propellant consumed (MCINNES, 1999). Clearly the sail system does not have any propellant to be consumed and in a mathematical point of view the specific impulse would be infinite, but that is only true if we had a infinite time mission. Since we have a finite time mission, the specific impulse is finite but it is not calculated as it is to chemical rockets. Let's take a look at the 'internal' efficiency $\eta_{int}$, which in this case is the irradiance that gets to the sail compared with the total irradiance that could reach the sail without jitter;

$$\eta_{int} = \frac{Irradiance\ reaching\ the\ sail}{Irradiance\ 'produced/available'} = 0.2181266 = 21.81\% \tag{3.7}$$

This internal efficiency indicate the effectiveness of the conversion energy input to the propulsion device and can be changed by using the system in much lower satellite altitude (a common altitude for others cubesats is around 300-400km), or change the location of the ground site emitting the laser to avoid the high density atmosphere (SUTTON, 2010).

The internal efficiency of a chemical rocket is defined as (for reference);

$$\eta_{int} = \frac{Kinect\ power\ in\ jet}{available\ chemical\ power} \tag{3.8}$$

and the typical balance for a chemical propulsion systema is shown in the figure bellow;
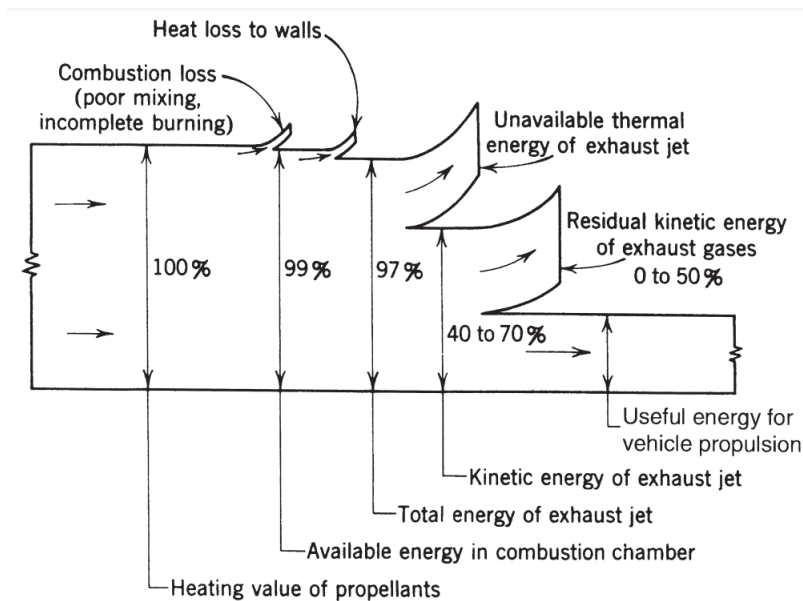


Figure 19 – Typical balance for chemical rocket shown in the book (SUTTON, 2010)

It shows that for a chemical rocket engine a typical internal efficiency is around 70%, and the same diagram can be adapted for the laser beam propulsion system;
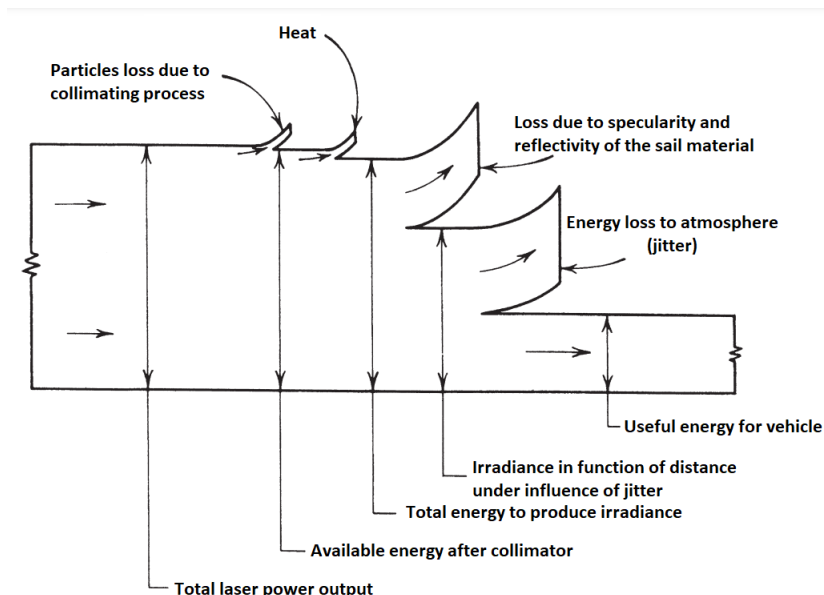


Figure 20 – Typical balance for laser beam (SUTTON, 2010) Adapted

The laser beam diagram does not have percentages because it will depend on the mission, but taking the parameters in this work and comparing to other propulsion system typical values of internal efficiency we have;

| Engine Type | $\eta_{int}$ |
|---|---|
| Chemical Rocket | 0.70 |
| Nuclear Fission | 0.50 |
| Arc-electrothermal | 0.40 |
| Ion electrostatic | 0.65 |
| 1 Laser Beam (From Earth) | 0.21 |

The table only shows the typical internal efficiency of the system and it is known that other parameters have to be considered, but here is just a quick look at the efficiency of the system itself.

# 4 Conclusion

The use of laser beam as a type of propulsion is a real challenge for the future to come, but right now we can see that in the mathematical point of view it works. Results shows that the amount of influence a 100kW laser beam would have in a satellite at 700km (average) altitude is real, but really small as the atmosphere will reduce by 4.58 times the irradiance reaching the sail. The answer to the question *Does it work?* is yes. But *is it worth right now?* is the proper question for this type of propulsion. Almost everything in this calculation were near idealization, not considering the sail stabilization analysis, accurate solar flux, and other mission planning that might prove another points of interest.

It was expected to have exponential cumulative force behaviour on the force created by the daily input of laser and it didn't happen in this particular case, as the change is so little in the satellite altitude we can see that the amount of power each beam increase to the sail will be in the same order thus creating a linear cumulative behaviour rather than exponential. Surely enough if taken to the extreme of years applying laser, it would show exponential behaviour. This work has only considered using one structure to send the beam, but we have to consider more than just 1 structure sending the beam. As one structure is not enough to cause too much influence it might be enough to make adjustments and to enhance drag for any purpose. It is easy to imagine that if we distribute these structures in the world, at right locations, we could have a constant force reaching the satellite and in this scenario the amount of influence would be significant. Another way to maximize the power delivered is to use the laser outside atmosphere, either in orbit or in another place without high density atmosphere. Further studies require the use of physics simulation softwares to input more realistic parameters and check the orbit change with real time visualization.

These results were all considering a cubesat at 700km altitude, but it is not common for the majority of cubesats in orbit. It might be the case to consider this study in orbits at 300-350km altitude and check the influence of the beam, because this is the common altitude of cubesats sent to orbit and at this altitude these results can change a lot, therefore this work conclusion is particular to the case of a cubesat at 700km with the parameters mentioned earlier.

Laser-driven propulsion is not as new as one might think, but the sufficient power to achieve results near the ones treated in this work are the challenge for this kind of propulsion system. Here we used a laser power of 100kW, but this type of laser is not yet available for commercial use and this amount of power is only achieved in some

laboratories working to military forces around the world (MARMO, ). In the date of the making of this work there is already lasers with power near 60kW being produced to be used in military trucks as a defense system against missiles.

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# Appendix

# APPENDIX A – Sci-Lab codes

## A.1 Irradiance code

```
//p-power, d-diametro da abertura, Ra- distancia do objeto, w-wavelength
/*

p = 100000;
d = 0.3;
Ra = 1015000:1000:1215000; //Considerando slant range e nadir angulo
w = 1024e-9;

rak=Ra./1000; //converter para km
*/
alfa = 45;
r1= 0.88;
s= 0.94;
ef= 0.05;
eb= 0.55;
bf= 0.79;
bb= 0.55;
c = 299792458;
//w = 1427.2795;

a1=32;

ji = 3e-6;

//Irradiancia real, com a aproximacao considerando jitter
function irradiance1
    //handlers para pegar dados das tabelas
    t2_handler=get(t2,'string');
    p=evstr(t2_handler(2,2));
    d=evstr(t2_handler(3,2));
    w=evstr(t2_handler(4,2));
    t3_handler=get(t3,'string');
    Ra1=evstr(t3_handler(2,2));
```

```
Ra2=evstr(t3_handler(2,3));
Ra=Ra1:273.9726:Ra2;
rak=Ra./1000;
//

i1= ((%pi*p*d^2)./(4*(w^2).*Ra^2))*(1/((1+(%pi^2)/2)*(ji/(w/d))^2));


//PARAMETROS GRAFICO
/*   da=gda()
//titulo
da.title.text="";
da.title.foreground=3;
da.title.font_size=4
//x labels
da.x_label.text="Distance␣[km]";
da.x_label.font_style = 2;
da.x_label.font_size = 5;
da.x_label.foreground = 3;
da.x_location = "bottom";
// y labels
da.y_label.text="Irradiance␣[W/m^2]";
da.y_label.font_style = 2;
da.y_label.font_size = 5;
da.y_label.foreground = 3;
da.y_location = "left";
da.thickness = 2;
da.foreground = 1;
da.zoom_box=[];
*/
    //Eq. 4.39
    R3_W = [cos(RA) sin(RA) 0; -sin(RA) cos(RA) 0; 0 0 1];


    //Eq. 4.40
    R1_i = [1 0 0; 0 cos(incl) sin(incl); 0 -sin(incl) cos(incl)];


    //Eq. 4.41
    R3_w = [cos(w) sin(w) 0; -sin(w) cos(w) 0; 0 0 1];


    Q_pX = R3_W'*R1_i'*R3_w';
```

```
/////////////// FOR AS E ACELERACOES////////////////////

p=i1/c;

     fn1=p*a1*((1+r1*s)*cosd(alfa)^2 + bf*(1−s)*r1*cosd(alfa)...
     +(1−r1)*((ef*bf−eb*bb)/ef+eb)*cosd(alfa));
     ft1=p*a1*(1−r1*s).*cosd(alfa).*sind(alfa);
     acn1=fn1/5;
     act1=ft1/5;




//////////////////POSICAO E VELOCIDADE PERIFOCAL/////////////

coe = [h, e, RA*deg, incl*deg, w*deg, TA*deg];
h = coe(1);
e = coe(2);
RA = coe(3);
incl = coe(4);
w = coe(5);
TA = coe(6);

rp = ((h^2)/mu)*(1/(1+e*cos(TA)))*(cos(TA).*[1;0;0] +sin(TA).*[0;1;0]);
vp= (mu/h)*(−sin(TA)*[1;0;0]+(e+cos(TA)*[0;1;0]));




////////////// DELTA V //////////////////////////////////////

t4_handler=get(t4,'string');
t=evstr(t4_handler);
vp1 =[];
zero=zeros(1,731);
deltaVn=(acn1*t)/1000;
deltaVt=(act1*t)/1000;
vectordelta=[deltaVn;deltaVt;zero];
vp1=vp;
for i =1:1:731
```

```
     auxvp=vp1+vectordelta (: , i );
     vpmatrix (: , i)=auxvp ;
     vp1=vpmatrix (: , i );
end


///////////////// ALTITUDE SATELITE (GERAL)/////////



for i =1:1:731
         vleo (: , i)=norm( vpmatrix (1:3 , i ))
end

for i =1:1:731
     z (: , i)=(mu/ vleo (1 , i )^2)−6378.137
end
zaux=z +6378.137;
for i =1:1:731
         T (: , i)=2∗%pi∗ sqrt ((zaux (: , i )^3)/mu);
         h1 (: , i)=(2∗%pi∗zaux (: , i )^2)/T (: , i );
         rpmatrix (: , i)=((h1 (: , i )^2)/mu)∗(1/(1+e∗cos (TA)))∗( cos (TA) . ∗[1;0;0] +s
         Rzaomatrix (: , i)=Q_pX∗rpmatrix (: , i );
end


////////////////////////////////////////////////////////


////////////////////CALCULO DA FORÇA DE ARRASTO//////////////
omega=[0 0 0.0000729]; // velocidade angular
rho =3.91e−14; // densidade 700km
//aux=cross (omega ,R); // auxiliar
for i =1:1:731 aux (: , i)=cross (omega ' ,Rzaomatrix (: , i ))  end ;
V=V∗1000;  // passando para m/s
for  i =1:1:731
vinf (: , i)=V'−aux (: , i );
end
for i =1:1:731
fd (: , i)=−1/2∗2.2∗rho ∗32∗ vinf (: , i )^2;
dragac (1 , i)=norm( fd (: , i ))/5;
end
```

```
//////////////////////ORBIT␣LIFETIME///////////////////////////
SH=90.5;␣//Scale␣height
P1p=0.00075186;//T␣ponto,␣taxa␣de␣mudan a␣do␣periodo
for␣i=1:1:731␣zalt=norm(Rzaomatrix(:,i));␣end

deff('L=f(T,zalt)','L=(3*SH*(T/60))/(2*zalt*P1p)');
dias=1:0.5:366;
figure␣(1);
␣␣da=gda();
//titulo
da.title.text="";
da.title.foreground=3;
da.title.font_size=4
//x␣labels
da.x_label.text="Days";
da.x_label.font_style␣=␣2;
da.x_label.font_size␣=␣5;
da.x_label.foreground␣=␣3;
da.x_location␣=␣"bottom";
//␣y␣labels
da.y_label.text="Lifetime␣(Days)";
da.y_label.font_style␣=␣2;
da.y_label.font_size␣=␣5;
da.y_label.foreground␣=␣3;
da.y_location␣=␣"left";
da.thickness␣=␣2;
da.foreground␣=␣1;
da.zoom_box=[];
plot(dias,f(T,zalt))

P1=98.542086:7.5186E−4:98.7451;
P1=gsort(P1,'g','d');
dias2=1:1:271;
L1=(3*SH*P1)/(2*a*P1p);

//////////////////VOLTAR␣PARA␣R␣e␣V///////////////////////
```

```
␣␣␣␣␣Rf=Q_pX*rpmatrix(:,731);
␣␣␣␣␣Vf=Q_pX*vpmatrix(:,731);



//////////////////////GRAFICOS////////////////////
figure(2);
␣␣plot(rak,i1);



␣␣[fn␣acn␣ft␣act␣i␣dVn␣dVt␣vpmatrix␣vleomedia␣rleomedia␣fd␣dragac␣rpmatrix␣T␣
␣␣i1,deltaVn,deltaVt,vpmatrix,vleo,z,fd,dragac,rpmatrix,T,fd,dragac,vinf,Rzad

endfunction
```

## A.2   Normal Force

```
alfa = 0: 1 :90;
r1= 0.88;
s= 0.94;
ef= 0.05;
eb= 0.55;
bf= 0.79;
bb= 0.55;
c = 299792458;
w = 1427.2795;
a=32;
p= w/c;
da=gda()
//titulo
da.title.text="";
da.title.foreground=3;
da.title.font_size=4
//x labels
da.x_label.text="Angle␣alpha␣[deg]";
da.x_label.font_style = 2;
da.x_label.font_size = 5;
da.x_label.foreground = 3;
da.x_location = "bottom";
// y labels
da.y_label.text="Normal␣Force␣[N]";
```

```
da.y_label.font_style = 2;
da.y_label.font_size = 5;
da.y_label.foreground = 3;
da.y_location = "left";
da.thickness = 2;
da.foreground = 1;
da.zoom_box=[0,0, 80,2.3e-4];
function [fn]=fnormal(r1,s,alfa,bf,bb,ef,eb,p,a)
    fn=p*a*((1+r1*s)*cosd(alfa)^2 + bf*(1-s)*r1*cosd(alfa)+(1-r1)*((ef*bf

    plot(alfa,fn);
endfunction
```

## A.3   Transverse Force

```
alfa = 0: 1 :90;
r1= 0.88;
s= 0.94;
c = 299792458;
w = 1427.2795;
a=32;
p= w/c;
da=gda()
//titulo
da.title.text="";
da.title.foreground=3;
da.title.font_size=4
//x labels
da.x_label.text="Angle␣alpha␣[deg]";
da.x_label.font_style = 2;
da.x_label.font_size = 5;
da.x_label.foreground = 3;
da.x_location = "bottom";
// y labels
da.y_label.text="Transverse␣Force␣[N]";
da.y_label.font_style = 2;
da.y_label.font_size = 5;
da.y_label.foreground = 3;
da.y_location = "left";
da.thickness = 2;
```

```
da.foreground = 1;
//da.zoom_box=[0,0, 90,0.00000000045];


function [ft]= ftrans(p,a,r1,s,alfa)
    ft=p*a*(1-r1*s).*cosd(alfa).*sind(alfa);


    plot(alfa,ft);
endfunction
```

## A.4    Getting **r** and **v** vectors from orbital elements

```
a = 7078.137;
// Parametros para o calculo



function RVcoe
    //mu - parametro gravitacional
    //coe - elementos orbitais [h e RA incl w TA]
    //h - magnitude do vetor H (km^2/s)
    //incl - inclinacao da orbita (rad)
    //RA - ascensao direita do no de ascensao(rad)
    //e - excentricidade (magnitude do vetor E)
    //TA - anomalia verdadeira (rad)
    //w - argumento do perigeu (rad)
    //R3_w - matriz rotacao em Z pelo angulo w
    //R1_i - matriz rotacao em X pelo angulo i
    //R3_W - matriz rotacao em Z pelo angulo RA
    //Q_pX - matriz de tranformacao do perifocal para o plano geocentrico equ
    //rp - vetor posicao no plano perifocal [km]
    //vp - vetor velocidade no plano perifocal [km/s]
    //r - vetor posicao no plano geocentrico equatorial [km]
    //v - vetor velocidade no plano geocentrico equatorial [km/s]
    coe = [h, e, RA*deg, incl*deg, w*deg, TA*deg];
    h = coe(1);
    e = coe(2);
    RA = coe(3);
    incl = coe(4);
    w = coe(5);
    TA = coe(6);
```

```
    //Eq. 4.37 e 4.38 (rp e vp sao vetores coluna)
    rp1 = ((h^2)/mu)*(1/(1+e*cos(TA)))*(cos(TA).*[1;0;0] +sin(TA).*[0;1;0
    //vp1 = (mu/h)*(-sin(TA))*[1;0;0] + (e+cos(TA))*[0;1;0];
    vp1= (mu/h)*(-sin(TA)*[1;0;0]+(e+cos(TA)*[0;1;0]));
    [rp vp]=return (rp1,vp1);
endfunction
```

## A.5   Getting orbital elements from **r** and **v** vectors

```
    //elementos orbitais do vetor de estado
mu=398600;
deg = 180/%pi;


a = 7078.137;
//R=[-5128.9029  4864.3875  10.430225];
//V=[-4.7128079  -4.9906925  3.057051];<- primeiro tcc
//V=[-5.1200461  -0.6831624  1.7947233];<- errado
// Antes de aplicar os deltaV's
// rp= [4884.8339 -5109.4439 0]
// vp= [5.4242129 0.6929511 0]


// Depois de aplicar os deltaV's
// Vn=0.00000265 [km/s]
// Vt=0.00000042 [km/s]
// depois de 90 dias aplicado duas vezes por dia
// Vn=0.000477 [km/s]
// Vt=0.0000756 [km/s]
// rp= [4884.8339  -5109.4439  0]
// vp= [5.4242885  0.6929538  0]


function coeRV


//mu - parametro gravitacional (km^3/s^2)
//R - vetor posicao no frame geocentrico equatorial (km)
//V - vetor velocidade no frame geocentrico equatorial (km)
//r,v - magnitude dos vetores R e V
//vr - velocidade radial (km/s)
//H - vetor momento angular (km^2/s)
//h - magnitude do vetor H (km^2/s)
//incl - inclinacao da orbita (rad)
```

```
//N − vetor da linha do node (km^2/s)
//n − magnitude do vetor N
//cp − produto [cross] dos vetores N e R
//RA − ascensao direita do nodes de ascensao(rad)
//E − vetor excentricidade
//e − excentricidade (magnitude do vetor E)
//eps − numero parametro para considerar que a excentridade e zero
//(abaixo desse numero a excentricidade e 0 para o calculo)
//w − argumento do perigeu (rad)
//TA − anomalia verdadeira (rad)
//a − eixo semi−maior (km)
//coe − vetor com os elementos orbitais [h e RA incl w TA a]
rv_handler=get(t1,'string');
R=evstr(rv_handler(2,2:4));
V=evstr(rv_handler(3,2:4));

eps = 1.E−10;
r = norm(R);
v = norm(V);
vr = R*V'/r;
H = cross(R,V);
h = norm(H);
//Equacao 4.7
incl=acos(H(3)/h);
//Equacao 4.8
N = cross([0 0 1],H);
n = norm(N);
//Equecao 4.9
if n~=0 then
     RA=acos(N(1)/n);
     if N(2)<0 then
          RA=2*%pi − RA;
     end
else
     RA = 0;
end

//Equacao 4.10
E = (1/mu)*((v^2 − mu/r)*R − r*vr*V);
```

```
e = norm(E);

//Equacao 4.12 (caso da excentricidade ser 0 e=0)
if n~=0 then
    if e>eps then
        w=acos((N*E'/n/e));
        if E(3) < 0 then
            w = 2*%pi - w;
        end
    else
        w=0;
    end
else
    w=0;
end

//Equacao 4.13a (caso da excentricidade ser 0 e=0)
if e>eps then
    TA=acos(E*R'/e/r);
    if vr < 0 then
        TA = 2*%pi - TA;
    end
else
    cp = cross(N,R);
    if cp(3) >= 0 then
        TA = acos(sum(N.*R)/n/r);
    else
        TA = 2*%pi - acos(sum(N.*R)/n/r);
    end
end

//Equacao 2.61 (a<0 para a hyperbola)
//a = (h^2)/mu/(1-e^2);

coe = [h, e, RA*deg, incl*deg, w*deg, TA*deg];

messagebox(["Parametro Gravitacional (km 3/s 2) ="string(mu)...
            "Momentum Angular (km 2/s) ="string(coe(1))...
            "Excentricidade ="string(coe(2))...
```

```scilab
                "Right ascension (deg) ="string(coe(3))...
                "Inclinacao (deg) ="string(coe(4))...
                "Argumento do perigeu (deg) ="string(coe(5))...
                "Anomalia Verdadeira (deg) ="string(coe(6))...
                "Eixo Semi-Maior (km): ="string(a)]...
                ,"Coeficientes Orbitais")


    h = coe(1);
    e = coe(2);
    RA = coe(3);
    incl = coe(4);
    w = coe(5);
    TA = coe(6);


    //Eq. 4.37 e 4.38 (rp e vp sao vetores coluna)
//    rp1 = ((h^2)/mu)*(1/(1+e*cos(TA)))*(cos(TA).*[1;0;0] +sin(TA).*[0;1;0]);
//    vp1 = (mu/h)*(-sin(TA))*[1;0;0] + (e+cos(TA))*[0;1;0];

[h e RA incl w TA a R V] = return(h,e,RA,incl,w,TA,a,R,V);

endfunction
```

## A.6   Simple interface to calculate everything

```scilab
//p=evstr(x_dialog('Valor da potencia [W]','100000'));
//d=evstr(x_dialog('Tamanho da abertura do laser [m]','0.3'));
//w=evstr(x_dialog('Valor do comprimento de onda [wavelength] [m]','100000'));
//Ra1=evstr(x_dialog('Distancia inicial [m]','1015000'));
//Ra2=evstr(x_dialog('Distancia final [m]','1215000'));
exec('Irradiance0.sci',0);
exec('coeRV.sci',0);
exec('RVcoe.sci',0);
/*exec('forcasdePrad.sci',0);
exec('deltaV.sci',0);
*/
m1=["","X","Y","Z";"R","","","";"V","","",""];
m2=["","";"Potencia","";"Abertura","";"Wavelength",""];
m3=["","Inicial","Final";"[metros]","",""];
/////////////////
```

```
f = createWindow ();
f.axes_size = [500  500];
f.figure_name = "Dados";


// TITULOS E TEXTOS //////////////////////////////////////////
text1=uicontrol(f, ...
    "style", "text", ...
    "string", "␣State−Vectors␣RV", ...
    "position", [60  460  85  20]);


text2=uicontrol(f, ...
    "style", "text", ...
    "string", "␣Parametros␣do␣laser", ...
    "position", [0  360  100  20]);


text3=uicontrol(f, ...
    "style", "text", ...
    "string", "␣Slant␣Range", ...
    "position", [145  360  100  20]);


text3=uicontrol(f, ...
    "style", "text", ...
    "string", "Tempo␣de␣aplica    o␣do␣Laser␣[Segundos]", ...
    "position", [0  260  200  20]);


/*
text3=uicontrol(f, ...
    "style", "text", ...
    "string", "For as␣e␣Acelera    o", ...
    "position", [250  230  200  20]);
*/
//BOTOES 'CALLBACK' //////////////////////////////////////////
pb1=uicontrol(f, ...
    "style", "pushbutton", ...
    "string", "Calcular␣Coeficientes", ...
    "position", [210  410  150  40],...
    "callback_type",2,...
    "callback", "coeRV");
```

```
pb2=uicontrol(f, ...
    "style", "pushbutton", ...
    "string", "Calcular", ...
    "position", [0 190 150 40], ...
    "callback", "irradiance1");

/*
pb3=uicontrol(f, ...
    "style", "pushbutton", ...
    "string", "Calcular", ...
    "position", [250 190 150 40], ...
    "callback", "face1");

pb4=uicontrol(f, ...
    "style", "pushbutton", ...
    "string", "Calcular", ...
    "position", [0 160 150 40], ...
    "callback", "deltav");
*/

// TABELAS ////////////////////////////////////////////////////
t1=uicontrol(f, ...
    "style", "table", ...
    "string", m1, ...
    "position", [0 400 200 59]);

t2=uicontrol(f, ...
    "style", "table", ...
    "string", m2, ...
    "position", [0 300 100 61]);

t3=uicontrol(f, ...
    "style", "table", ...
    "string", m3, ...
    "position", [120 318 150 43]);

t4=uicontrol(f, ...
    "style", "edit", ...
    "position", [0 230 60 30]);
```

```
/////////////////

mu=398600;
deg = 180/%pi;
//handler de fun    es

///////////////////////////////////////////////////////////////
// Primeiro passo
// pegar o valor atual do vetor-estado
//R=[-5128.9029 4864.3875 10.430225];
//V=[-4.7128079 -4.9906925 3.057051];

// passar para o plano perifocal
    // pegando dados da orbita
```