



POINT CLOUD FILTERING

ANDRÉ NÚNCIO DE OLIVEIRA SOL

**TRABALHO DE CONCLUSÃO DE CURSO EM ENGENHARIA ELÉTRICA
DEPARTAMENTO DE ENGENHARIA ELÉTRICA**

**FACULDADE DE TECNOLOGIA
UNIVERSIDADE DE BRASÍLIA**

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FILTRAGEM DE POINT CLOUDS

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TRABALHO DE CONCLUSÃO DE CURSO SUBMETIDA AO DEPARTAMENTO DE ENGENHARIA ELÉTRICA DA FACULDADE DE TECNOLOGIA DA UNIVERSIDADE DE BRASÍLIA COMO PARTE DOS REQUISITOS NECESSÁRIOS PARA A OBTENÇÃO DO GRAU DE ENGENHEIRO ELETRICISTA.

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RESUMO

Título: Point Cloud Filtering

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Orientador: Prof. Daniel Chaves Café

Programa de Pós-Graduação em Engenharia Elétrica

Brasília, 17 de fevereiro de 2020

Este trabalho realiza uma revisão dos diversos métodos existentes para a filtragem de nuvem de pontos. Com o foco em reconstrução de objetos de pequeno porte escaneados a laser. O escâner utilizado foi desenvolvido na própria universidade e é composto por um sensor de distância a laser VL53L0X, baseado em tecnologia Time of Flight (ToF), e dois motores de passo, um para mover o sensor e outro para o objeto.

Apresentam-se cinco princípios de filtragem: filtragem de forma estatística, filtragem baseada na vizinhança de pontos, filtragem por projeção em superfície, técnicas de processamento de sinais e por meio equações diferenciais parciais. Os métodos foram aplicados Moving Least Squares, Operador Laplaciano, Operador de Taubain e simplificação de nuvem com o auxílio do software MeshLab.

Testou-se duas amostra de nuvem de pontos, uma criada por computador e outra amostrada pelo escâner 3D do laboratório. Variou-se os parâmetros dos filtros e por fim realizou-se uma análise qualitativa de ambos os resultados.

Palavras-chave: Nuvem de Pontos, Filtragem de Dados, Escâner 3D, Reconstrução de Objetos.

ABSTRACT

Title: Filtragem de Point Clouds

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This work reviews several filtering methods for point cloud filtering. The main objective is to recreate in a virtual environment small objects sampled from a 3D scanner. The scanning device used was developed at the university. It is made of a laser ranging sensor, called VL53L0X, that uses the time of flight (ToF) technology, and two stepper motors to move the sensor and the object.

Five filtering principles are shown: statistical-based filtering, neighborhood-based filtering, projection-based, signal processing methods and partial differential equations based. With the use of MeshLab software tested the filtering methods. At the end the reconstruction of smooth surfaces from the scanner samples was possible. The Moving Least Squares Algorithm, Laplacian operator, Taubain operator and Point Simplification were applied with the MeshLab software.

Two point clouds were tested. One created by a computer and the other sampled from a real object by the 3D scanner. Varying the filters parameters we tested the quality gain after the usage.

Keywords: Point Cloud, Data Filtering, 3D Scanner, Object Reconstruction.

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LIST OF ACRONYMS AND ABBREVIATIONS

CAGD	Computer Aided Geometric Design. 4
CLOP	Continuous LOP. 15
DFT	Discrete Fourier Transform. 17
EAR	Edge Aware Upsampling. 10
FLOP	Feature Preserving LOP. ix, 15
FT	Fourier Transform. 17
GMM	Gaussian Mixture Technique. 15
IRLS	Reweighted Least Squares. 9
KDE	Kernel Density Estimate. 15, 18
LADAR	Laser Detection and Ranging. 4
LOP	Parametrization free local projection. ix, 14, 15
MLS	Moving Least Squares. ix, x, 15, 16, 19, 20, 22, 23, 26, 27, 29, 30, 32
NURBS	Non-Uniform Rational B-Splines. 4
PCA	Principal Component Analysis. ix, 7–10
PCL	Point Cloud Library. 20
PDE	Partial Differential Equation. viii, 6, 18
PLY	Polygon File Format. 20
SDA	Scattered Data Approximation. 17
STLOP	Spatial Temporal LOP. 15
ToF	Time of Flight. viii, 2, 3, 5
WLOP	Weighted LOP. ix, 14, 15, 18

1 INTRODUCTION

In 2018, the Rio de Janeiro National Museum, the biggest Natural History Museum in Latin America [6], suffered a fire that destroyed cultural and historical artifacts. The unique, irrecoverable items that were lost represented the legacy and history of our past. Such a great loss strive people to find better ways to preserve such heritage. In order to preserve these objects in a virtual space we propose a 3D scanner capable of generating a virtual afterimage of a existing object. Jeremy [7] started the project in 2018 by doing a technological review for suitable parts and equipment and Ricardo [8] continued it in 2019 by designing the final mechanical parts and assembly the 3D scanner, later on his work he scanned a few objects, which a used in this work.

The idea of a virtual museum is not new. The advancements in 3D cameras propelled new forms of cataloging and sharing objects that serve as records of history and culture. However, this technology proved to be too costly and requires skilled labor to operate, which goes against the idea of accessibility. The main focus of this project is the implementation of a low-cost manufacture and easy operation 3D scanner.

The goal is to provide a low-cost, easy to operate, 3D scanner to be used by professionals and hobbyists that crave to reproduce real world objects in a virtual 3D environment. Some of the possible uses are: Preserve museum artifact in a digital form, reverse engineer a mechanical part for later reproduction, make it easy to create to create virtual environments with real world objects and so on.

1.1 PROJECT HISTORY

The project started in 2018 with a technology review for the selection of possible sensors and materials [7]. In order for the proposed scanner to get the upperhand against the ones available on the market it must be cheaper. The focus of the project was set at low-cost 3D scanner. The parts to build it chosen accordingly. Taking on from Jeremy's work Ricardo [8] built the first prototype, as shown Fig 1.1, using the chosen laser ranging sensor and two stepper motors. Ricardo designed the mechanical parts missing from the project and 3D printed them. In his work he characterized the sensor and assembled the 3D scanner. The prototype is capable of scanning small objects.

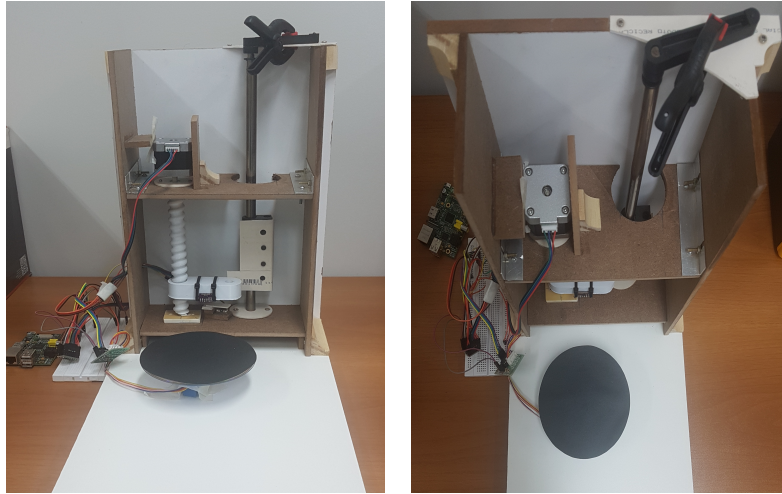


Figure 1.1 – Prototype front view at the left. Top view at the right

Reproducing an object to the digital world is acquiring the geometrical proprieties of its surface. The best way found by Jeremy's work is measuring the distance of the surface from a certain referential. In the variety of sensors capable of acquiring distances three were discussed: Magnetic, Sound and Optic sensors.

The magnetic sensors are able to measure distance from metallic objects only, and therefore can't be used generically. sound and optic sensors both share the same principle, they use the ToF method. A signal is sent from an emitter and then reflects on the surface. The signal then returns to the receiver part of the sensor. The time that the signal travels from the emitter to the receiver is measured. The speed of the emitted signal is known, so the sensor is able to determine the distance from the reflection point and itself. Analyzing the propagation characteristics of the both options, the laser emitting proves to be more precise as light emission can be more focused than sound.

The mechanical part was designed inspired in 3D printed technology. A vertical rail guides the sensor and a rotatory base spin the object so it can be scanned, one point at a time. After several rotations and adjustments all the points collected are saved in a file. This file becomes the signal that represents the physical object.

Nonetheless these signals suffer a great amount of deterioration. Noise contamination and outliers corrupt part of the data. These artifacts come from several reasons, such as limitations of the sensors, inherent noise of the acquisition device, light or reflecting nature of the scanned object.

1.2 OBJECTIVES

The objective of this work is to provide a software solution to the inherent noise caused by the ToF light sensor of the low-cost 3D scanner. This work will focus on noise removal and outliers cropping.

1.2.1 Specific Objectives

- Study the visualization software for point clouds.
- Implement noise removal filters to the point cloud.
- Implement outlier cropping filters to the point cloud.
- Reconstruct point cloud surface.
- Compare results to the real objective.

1.2.2 Text Form

This work is divided in: Literature Review, Methodology, Results, Conclusion. Literature reviews presents the important concepts needed to understand the process and clarify the terminology used trough the work. Methodology show how the experiments were done and how to reproduce them. Results show the the final reconstructions and important features. Lastly, conclusion show the knowledge acquired by this work.

2 LITERATURE REVIEW

This chapter explains the terms and the techniques studied for this work. Start with the review on point clouds and the VL53L0X sensor used to acquire them. Then moves to 3D filtering and reviews the statistical-based filtering, neighborhood-based filtering, projection-based filtering, signal processing based techniques, partial differential equations based filtering and hybrid techniques.

2.1 POINT CLOUDS

Point Cloud is a new primitive representation of objects [9]. It consists of data points scattered in space, that forms the surface of an object. Essentially, points are samples of a 3D object. The points are three dimensional vectors that represent the spatial coordinates x , y , and z . Point Clouds are represented by a list of all points of an object. Different from pixels, vectors and polygons, point clouds have a more powerful representation capability and flexibility due to its own simplicity. It does not store or maintain the polygonal-mesh connectivity, making it a light digital representation of a 3D object. The data structure of the point clouds is basically a list of three dimensional positions that determine a point in space.

Such advantage makes it easier to process and handle as there is less overhead in algorithms and better performance. However this is not the only reason of its popularity. Many sensors deliver data in a format that is very close to a point cloud. Using a point cloud data format leverages the process of data acquisition as well.

Laser Detection and Ranging (LADAR) measure distance using light delay from the laser it emits. Several applications need this type of data to work, such as autonomous vehicles, object recognition and 3D scanning for digitization of real world objects. The raw data acquired by such techniques are the collection of points.

Other options for 3D surface representation are Non-Uniform Rational B-Splines (NURBS) [10] and polygonal meshes[11]. The former is very successful in Computer Aided Geometric Design (CAGD). NURBS was developed by automobile engineers with the intention of modeling car parts. It consists in parametric surfaces determined by tensors. The latter is very popular in rendering. It has more flexibility and precision than NURBS, but still fall behind in simplicity when compared with point clouds.

2.2 SENSOR VL53L0X

Manufactured by the STMicroelectronics the VL53L0X sensor is a ToF laser-ranging that can measure up to two meters, according to the manufacturer. After been tested it showed some particular behavior. It is heavily affected by the reflective and refraction characteristics of the target. The sensor is also affected by the color of the object it is measuring. Its aperture can measure points at closer distance and from afar at the same time, which generates noisy data.

2.2.1 Time of Flight (ToF) Technology

ToF is a measurement of the time taken for a signal to run a certain distance, in this case a electromagnetic wave is emitted by the sensor and it is reflected on the surface of the target, traveling back to the sensor and hitting the receiver module. Since the speed of light is constant in the air, it is possible to determine the distance of the target by the time delay of the two signals, the emitted and the received. The Fig 2.1 shows the schematic of the sensor function. The signal time is measured by the phase shift between emitted and received signals.

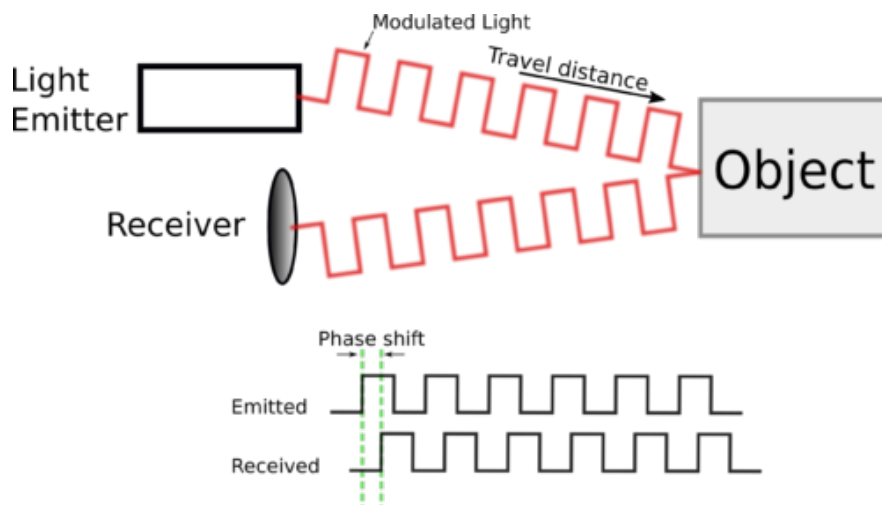


Figure 2.1 – ToF schematic

2.2.2 Low-Pass Characteristics

The sensor makes several measurements of the same point and then send the average value, which by itself classifies as a low-pass filter. However, the light emitted by the sensor also suffer diffraction alongside the path traveled, transforming the field of vision of the sensor into a cone instead of an ideal line. Thereby different points inside the sensor's circle of sight are measured as one, smoothing the surface locally.

2.2.3 Noise Corruption

Aside from the white noise typical of electronic devices, this sensor suffer from three particular noise sources: Cross-talk noise, Refractive and reflective proprieties and signal diffraction. The cross-talk noise is natural to the sensor installation. The manufacturer recommend using a translucent protective shield in front of the sensor's emitter and receiver, to protect it from dust and other particles capable of blocking it. The protective shield causes part of the signal to reflect on it, and cause different signals to reach the receiver. The reflection in the protective shield gives the impression that the object is closer than it actually is.

Secondly the refractive and reflective nature of the target directly weights in the measurement, depending on how translucent or the color of the object the laser might travel different paths before reaching back. Lastly the aperture of the sensor being a cone creates artifacts close to the edges, since it measures points that are faraway and close to the sensor at the same sampling window, when it averages these values it gets noisy points.

2.3 3D FILTERING

Filtering is a wide area of research and is usually the first step on any data processing pipeline[12]. Many techniques used for other data structures can be adapted for points. Point cloud filtering is in fact an adaptation of classical mesh, image and video filtering.

The adaptation of digital image algorithms is not direct, for three reasons [2]. Irregularity: point clouds do not require regular sampling like audio or images. In a point cloud different regions may have different point density. Shrinkage: the effect of moving points for lessening the energy on the point cloud, which moves the points closer to each other may cause the total volume of the cloud to decrease. At last Drifting: moving a group of points in one direction causes spatial displacement of regions.

The methods will be divided in groups, commonly used in the literature. Statistical-based, Neighborhood-based, Projection-based, Signal Processing Based, PDE Based and at last the Hybrid and Miscellaneous methods. These methods consist in using the geometrical and spatial properties of the three dimensional data points to properly reconstruct the surface.

Robustness in filtering means that the technique works well on noisy data with small errors and gross errors equals to outliers. [1]

2.3.1 Statistical-Based

Consider a point cloud as a set of samples that are attached to a real object. The adaptation of statistical techniques are suitable for point cloud processing. The correlation and point distribution along the signal are proper to statistical distributions.

A non parametric kernel density estimation scheme was proposed by Shall et al. [1]. The main idea is to find cluster centers. Points that better represent the surface around it. This method delivers an accurate approximation of the surface desired. Then use the mean shift[13] technique to detect local maxima. Clusters associated with the maxima are detected by a threshold. Despite being a good technique for noise attenuation sharp features are also smoothed like the texture on the statue hair as shown in Fig 2.2.

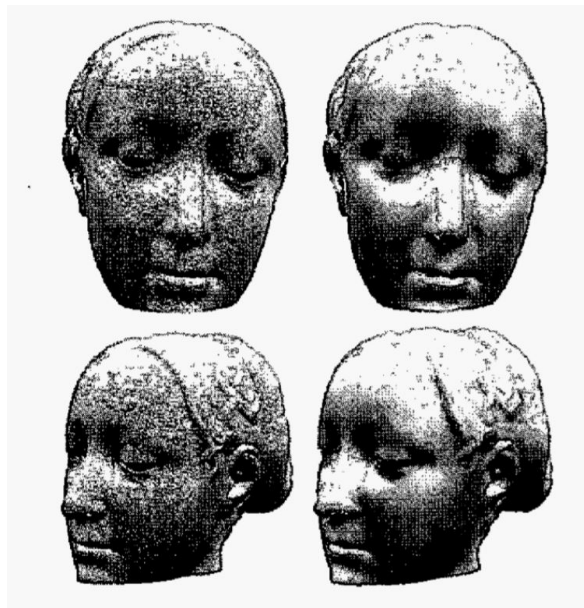


Figure 2.2 – First column the noisy data and the second column the filtered data[1]

Principal Component Analysis PCA[2] is another statistical-based method widely applied in data simplification and study. This technique is usually employed to reduce dimensions and bias cutback. Implementing PCA into a small group of points will return a matrix with three eigenvalues. Eigenvalues represents the best plane to fit those points. The first eigenvector represents the normal of the plane. Nevertheless the PCA is very sensitive to outliers as shown in Fig 2.3, so it is usually applied a variation of the PCA in the literature, weighting the point for feature preservation, shrinkage prevention and bias correction. Fig 2.4 shows the result of the proposed method by Leal et al.

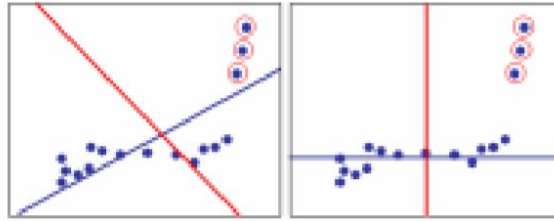


Figure 2.3 – Left: PCA without weighting. Right: with the proper weighting[2]. The red dots are outliers

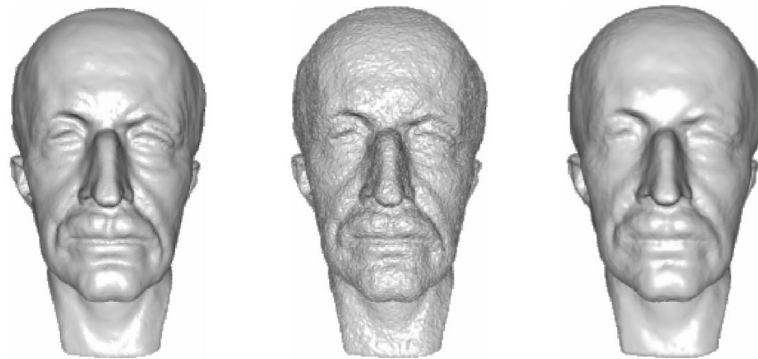


Figure 2.4 – Original Model, Noisy Model and Filtered Model[2]

One of the most canonical forms to solve reconstruction problems are the application of Bayesian Statistics. Jenke et al. [3] propose a technique for noise removal and scene reconstruction. Start with the assumption of a scene S made of n points. Delete some points leaving m points. Finally some random noise is injected into the remaining points, creating the measured scene D . To construct the estimated point cloud \tilde{S} , every m point of D is associated to an n point of \tilde{S} .

These points affect the a likelihood of scene \tilde{S} has ($P(\tilde{S}|D)$) in terms of its a priori probability $P(\tilde{S})$ and posteriori probability $P(D|\tilde{S})$. The remaining points ($n - m$) are placed near the m points with probability inversely proportional to the measurement sample density, since these only affect $P(\tilde{S})$. This method does not work well if there is already holes in the measured surface. Small holes are filled automatically, larger ones must be filled manually, as shown in Fig 2.5



Figure 2.5 – Scanned Data, Reconstruction, Manual Hole Filling, Final Rendering[3]

Kalogerakis et al. [4] show that points clouds can be filtered based on Interactively Reweighted Least Squares (IRLS) a framework based on computing the curvature of the point cloud through a maximum estimation of likelihoods. This method refines the shape of every point's neighborhood by weighting samples through a fitting error. The kernel used to read the neighborhood must automatically adapt to the surface, getting larger near irregularities, heavy noise and edges.

This method computes the point normal by estimating curvatures, showing better results than PCA-based methods. After that the global energy of the point cloud is minimized in order to denoise it, while extracting the curvatures of the discontinuities in order to give robustness against surfaces of any kind. The Fig 2.6 show how the curvature lines still follows the object even with noise and density irregularity of points.

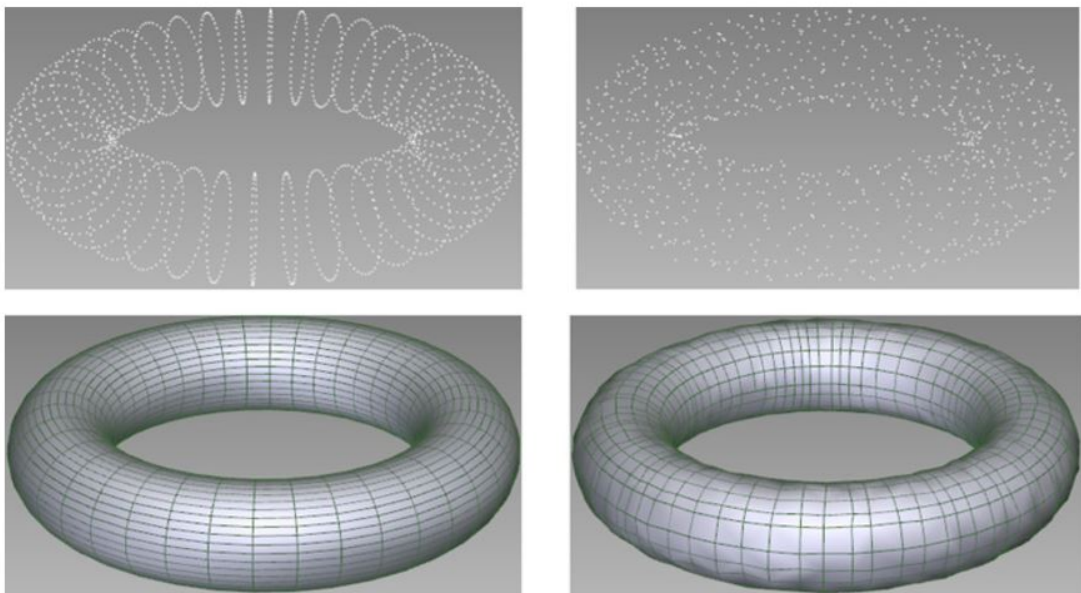


Figure 2.6 – Regularly sampled torus and Noisy random sampled torus[4]

At last, encouraged by sparse signal reconstruction and compressive sampling Avron et

al. propose L_1 -minimization techniques for point clouds [5]. Common objects can often be characterized by a small number of features even the geometrically complex ones. Utilizing L_1 -minimization for sparse signal reconstruction. A method is applied first solving for points normal orientations. This method achieves better result than PCA based. Solve for point positions as shown in Fig 2.7 based on surface smoothness. For better results the L_1 -minimization is reweighted to achieve better sparsity reconstruction [14]. With the adaptive weights the method can recover surfaces with less samples. This technique shows good result even for high textured objects.

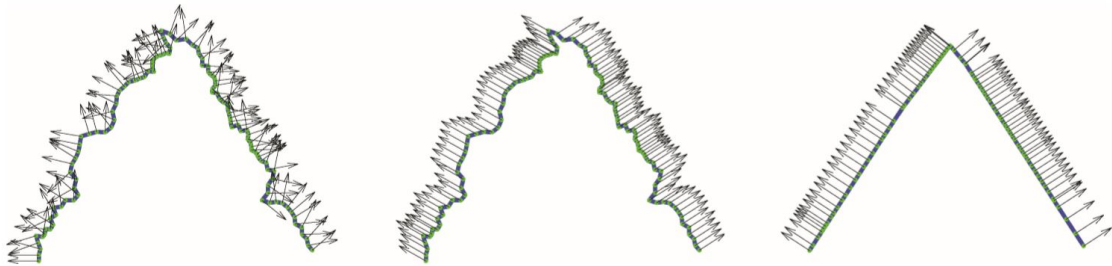


Figure 2.7 – Noisy V-shape, Normal Regularization and Point Position Regularization[5]

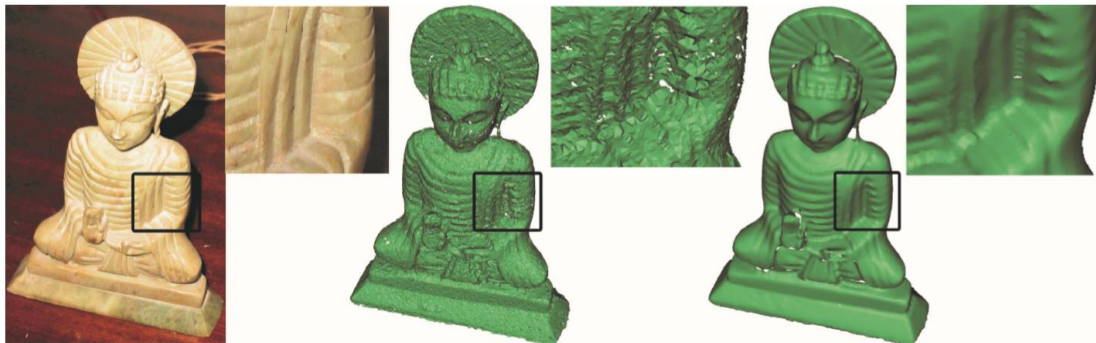


Figure 2.8 – Real Object, Scanned Object and Reconstitution[5]

Alongside the adaptation of L_0 -minimization for 2D images Sun et al [15] proposed an algorithm that outperform the former and enhances the Edge Aware Upsampling (EAR) algorithm. EAR is used to generate more points based on the existing ones for better rendering. As shown in Fig 2.9 this method can perceive sharper features.

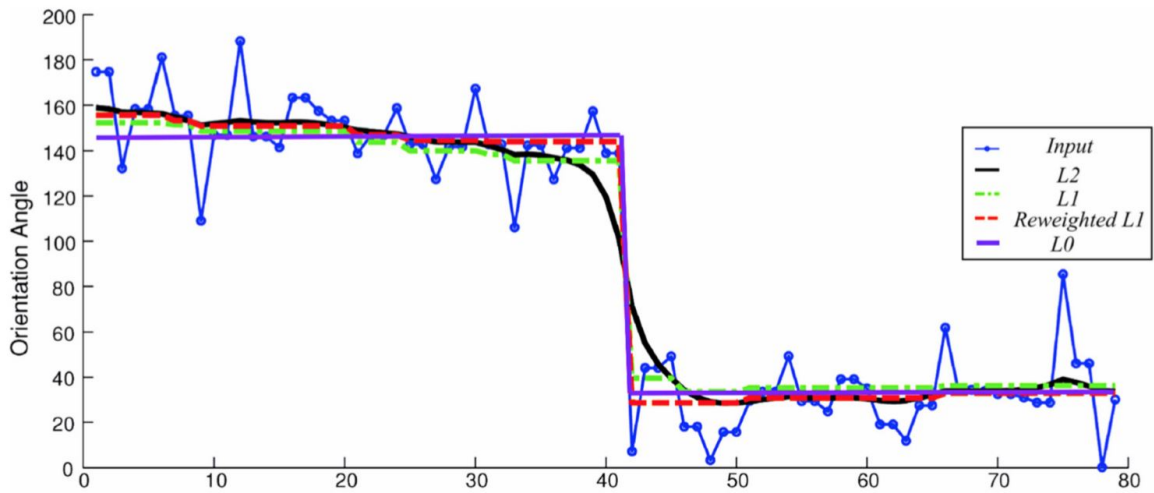


Figure 2.9 – Smoothing 1D signal using different norms[15]

2.3.2 Neighborhood-Based

Neighborhood filtering is the determination of a point position in account of similarity measures between it and the closest points. The point neighborhood can be determined by point positions, normal and regions.

The Mean Shift Filter [16] is a dynamic nonlinear filter. It achieves a high quality filtering while preserving discontinuities. As many techniques, this was also adapted from image processing field to 3D data. It is done by taking vertex normal and curvature as the range instead of pixel intensity and position as spatial component. Then the points are saved as a dataset based on these components. It is divided in small groups called patches. Find the local mode of each sample point and cluster points of similar local modes. Fig 2.10 shows the modes been separated by octree [17] method. This method uses octree separation for better performance. It is a very simple method but does not preserve sharp features.

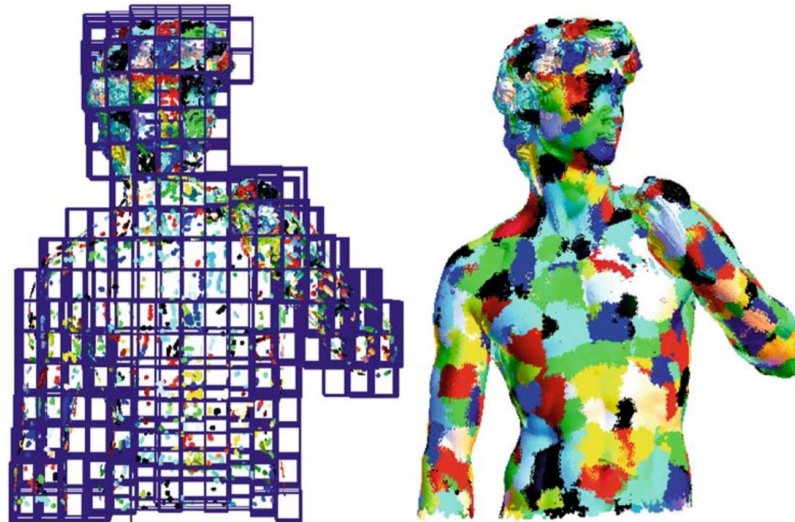


Figure 2.10 – Separation by octree [16]

Filters designed for range scans are heavily based on image algorithms, since its data structure mirrors the grid of an actual photograph. Bilateral filtering[18] is heavily used in such conditions, it is applied switching the pixel intensity by other measures [19]. Schall et al [20] designed a non-local neighborhood filter that determines the denoised position of a vertex as a weighted average of similar vertices in its vicinity. Comparing regions generate better results than comparing range and normals. For colored scans the color is used to weight on region selection for points since continuous surfaces tend to have the same color along itself [21].

Fig 2.11 shows the noise reduction of Shall's algorithm, the green images represent the contrast on the point cloud surface. After processing the sharper regions are limited to the sharper features.



Figure 2.11 – First Row is Noise data, Second is Bilateral Filtering and Third is Shcall et al method [20]

Considering the problem of low-quality data from range scans, a connectivity-based outlier detection method [22] was proposed. It is insensitive to the size of the chosen neighborhood. This technique generates better results on high noise regions.

Determining the correct neighborhood depends on where the point is. Several planes on the point neighborhood are selected and sorted properly, while keeping sharp features undistorted. This method designed is for scanning city scenes. Composed from big blocks of buildings. It may not work well on a more populated environment like forest and parks.

2.3.3 Projection-Based

Projection-based techniques are the ones which the final position of a point is determined by its projection on a reference. A good example is the point cloud skeleton algorithms, they grasp the simplest structure of an object. Based on L_1 -medial for data points. The work proposed on [23] creates a good reference to project data points. Fig 2.12 shows an example.



Figure 2.12 – Dinosaur model, Raw scan and skeleton generated [23]

Lipman et al. Proposes the LOP operator [24]. It uses a more primitive projection mechanism, but since it is not 2D it is more robust and goes well in complex settings. It is a projection operator rather than a filter and heavily used in preprocessing. Nonetheless it can be used to reduce noise and crop outliers. It uses the statistical tool L_1 Median to calculate the projection. Fig 2.13 shows the smoothness achieved by the LOP operator

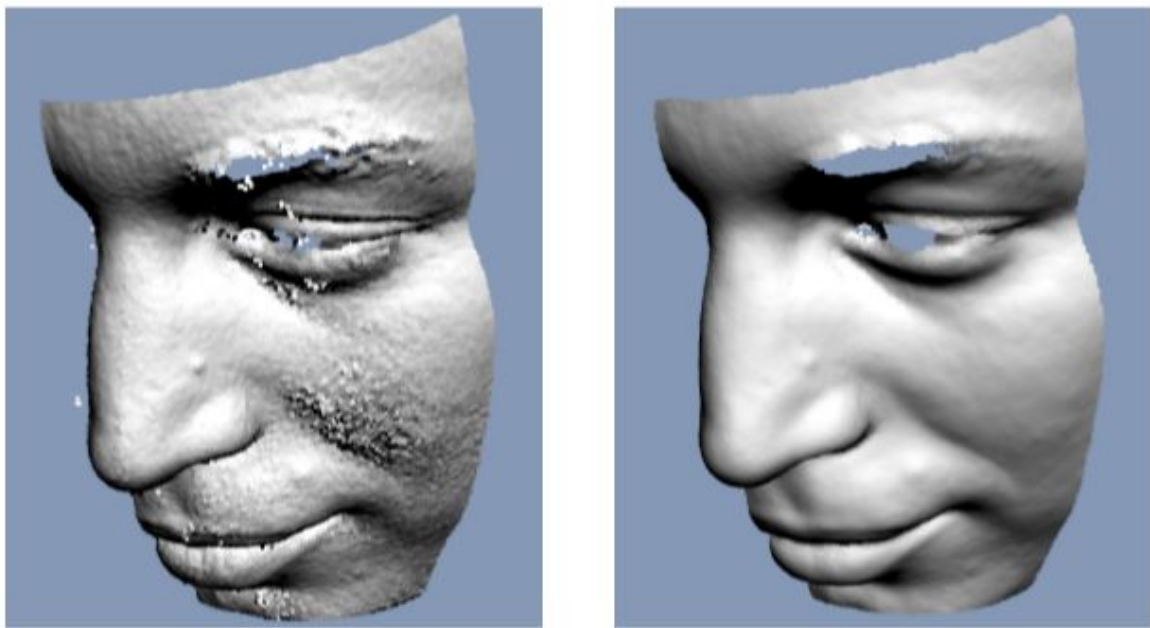


Figure 2.13 – Noisy data and LOP reconstruction [24]

However LOP algorithm doesn't work well on non-uniform distribution. Fig 2.14 shows the distribution of projections. The LOP projection mirror the data distribution, so they are projected non-uniformly. To overcome this barrier, the WLOP [25] was proposed. With a parameter that factor the point to a even distribution. This factor may cause the shrinkage of the point cloud if it is too large or cause it to diminish any denoise.

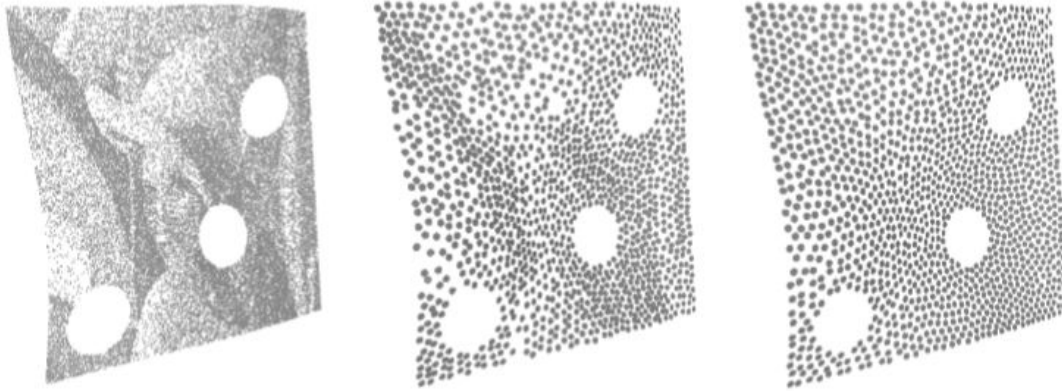


Figure 2.14 – Original data, LOP projection, WLOP projection [25]

Whereas these two last techniques have really good performance in noise removal, they don't do well in feature preserving. Sharp features are smoothed or distorted by those algorithms. With that in mind the FLOP was proposed [26] by Liao et al. It creates a feature preservation weight. First it is applied a bilateral-weighted local optimal projection for feature preserving. Then it takes both spatial and geometrical information. FLOP is based on Kernel Density Estimate (KDE) [27], an adaptive local parameter. The author even propose a filter for time-varying data Spatial Temporal LOP (STLOP).

Fig 2.15 shows the effect on redistributing the points. Sharper features require more points to be represented, by projecting without regard of region LOP and WLOP can not reconstruct sharp features well. FLOP keep more points on edges so it won't be distorted.

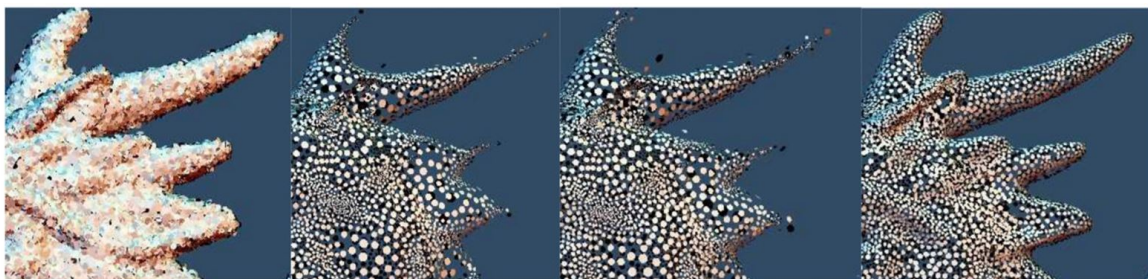


Figure 2.15 – Original data, LOP projection, WLOP projection, FLOP projection [26]

For better performance switch the point projection for gaussians, using Gaussian Mixture Technique (GMM) to WLOP creating Continuous LOP (CLOP) [28]. This method has better computer performance and generates smoother surfaces.

A different but very common use is the Moving Least Squares MLS [29], a process to find a local plane and project the point upon it. This method is very popular and already had been modified by different authors. The main reason is the fact that it is a low-pass filter that shrinks the point cloud and the nonlinear part of the process. Alexa et al [30] proposes

the substitution of the non-linearity of the MLS for an weighted average. Mederos et al [31] adapts statistical theory to improve MLS in order to preserve sharp features, applying a M-estimator. Fig 2.16 show the smoothness achieved and the texture preservation of this method.

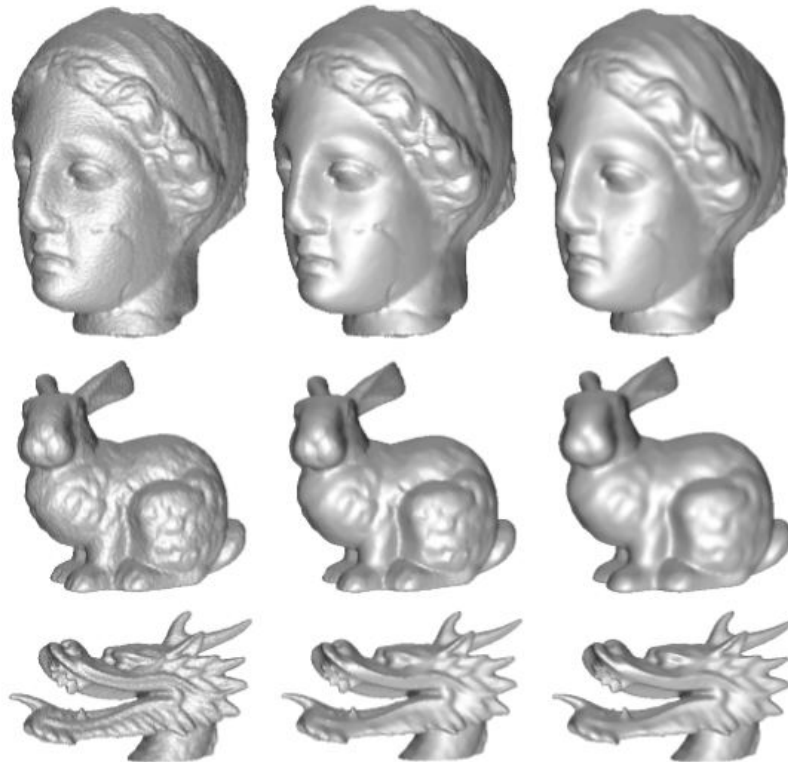


Figure 2.16 – Noisy Object, Improved MLS, Original MLS [31]

Employing the forward-search paradigm Fleishman et al [32] find the best neighbors to apply the MLS projection, providing great gain in edge preservation and noise filtering. As shown in Fig 2.17. The edges of the box are better represented. The MLS makes them rounded.

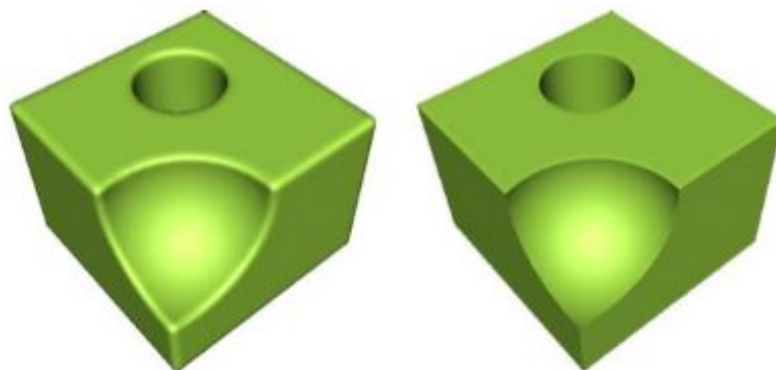


Figure 2.17 – Normal MLS and Fleishman's MLS 2.17

2.3.4 Signal Processing Based

Adapting the signal processing pipelines to point cloud processing results in fast applications which are ideal for real time analysis. The Laplacian operator transforms an optimization problem into a low-pass filter design problem [33]. Yet this method causes shrinkage. To solve this problem, proportional cylindrical rescaling is applied to the whole point cloud [34].

The result can be seen in Fig 2.18 where the left image shows the local rescaling and the right one shows global rescaling, the middle is the original point cloud. The local technique has less distortion when compared to the global approach.

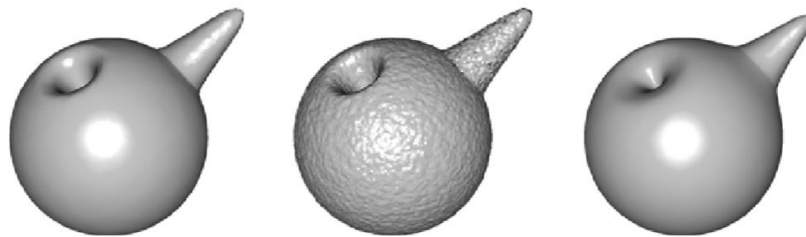


Figure 2.18 – Result of Laplacian Operator 2.18

Perhaps the most famous operation in signal processing is the Fourier Transform (FT). The FT has several problems when adapting to point clouds. FT requires global parametrization, regular sampling pattern and more spatial localization. In order to solve these issues the FT is applied in windows [35] that transform the irregular point cloud to sets of regularly sampled height maps. The pipeline goes as follows.

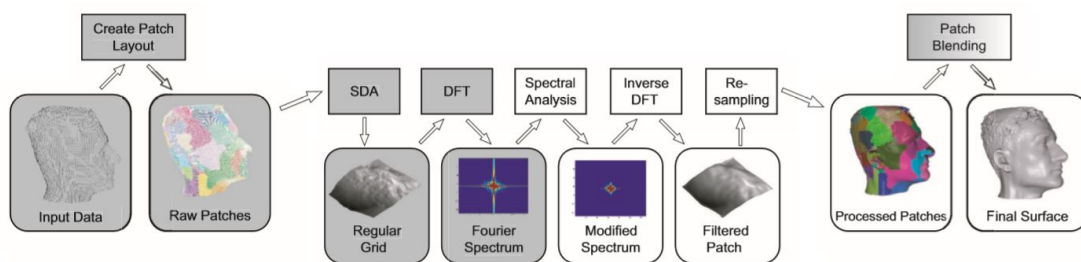


Figure 2.19 – Fourier Transform Pipeline [35]

The points are clustered into selected patches. For better reconstruction these patches keep the neighborhood information so they can be restitched back again. Then a fast Scattered Data Approximation (SDA) is used to populate the grid with missing points. Then Discrete Fourier Transform (DFT) is applied on the grid and filtering transformations on the frequency spectrum. At last the process is undone to retrieve the point cloud again.

2.3.5 PDE Based

PDE techniques is an important tool for computer vision and was also adapted to point clouds. PDE can only be applied onto meshes. Since point clouds surfaces are not meshes it can not be applied finite elements tools. To work around this problem Clarenz et al. [36] proposed the creation of a number of local finite elements matrices over small neighborhoods. The main idea is to combine points into surfaces and then solve the PDE problem. However, this technique can't keep sharp features. Another problem is the time consuming step of computing the partial equation properties.

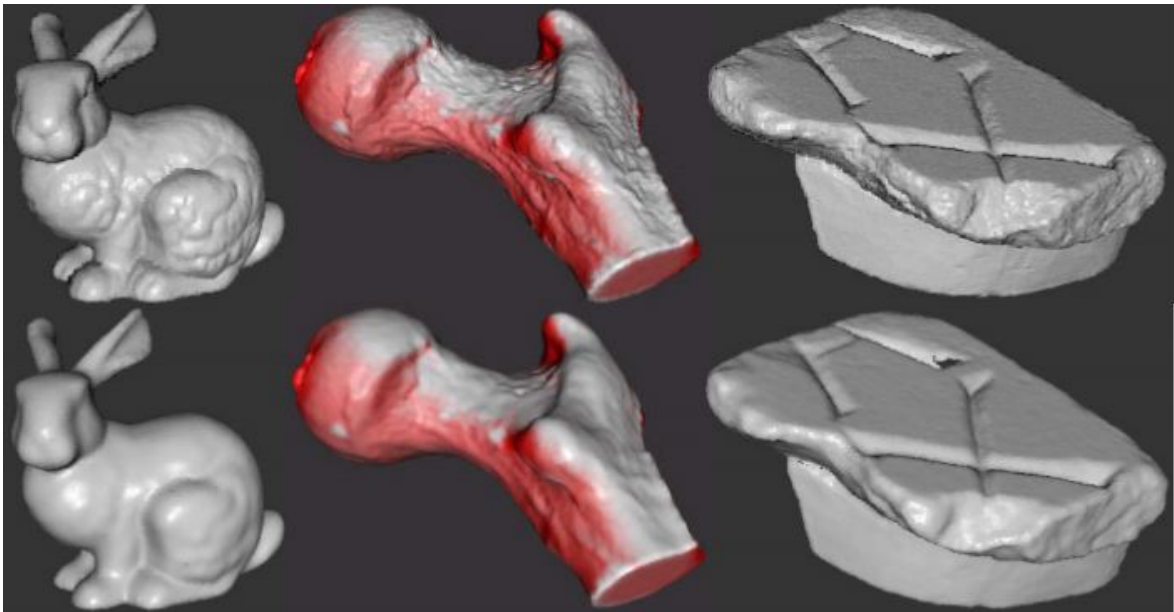


Figure 2.20 – First row original objects, second row filtered objects [36]

2.3.6 Hybrid and Miscellaneous

As noted, most of the shown applications involve several techniques, locally or globally. It is hard to solve a problem with only one technique. Combining WLOP and mean shift outlier removal [37] as shown in Fig 2.21 resulted in a very timing costly implementation and yet won't recover sharp features well. Another example is particle swarm estimation for a KDE and then applying bilateral filter [38] as shown in the Fig 2.21.

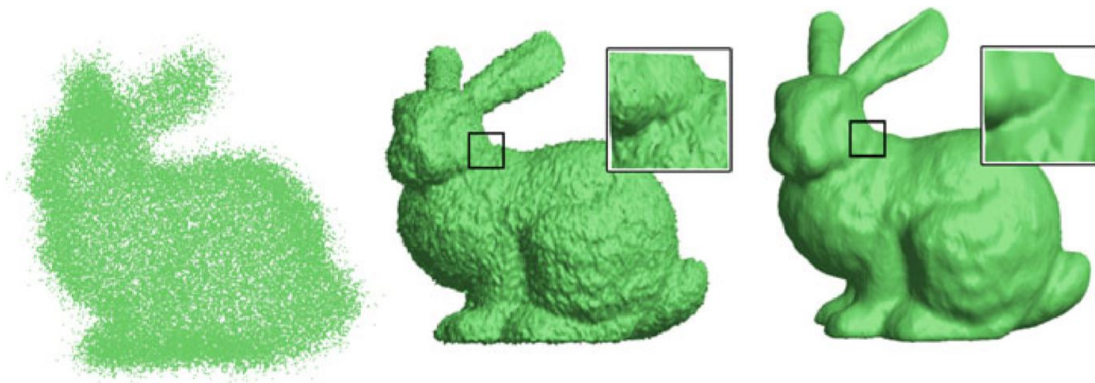


Figure 2.21 – Point Cloud, Noisy Object and Filtered Object [38]

Considering the initial state of development of the laboratory's 3D scanner this work chose the most general techniques. Techniques with easier implementation were prioritized from difficult implementation ones. From Projection-based methods the MLS was chosen, it has a good smoothing factor and doesn't deform smooth surfaces only sharp features. From Signal Processing the Laplacian and Taubain operators were chosen. This two methods are simple and present good results. Taubain operator has a scaling factor that prevents shrinkage. Lastly from Statistical-based filtering the Downsampling method was chosen. It is not a good method for sharp features but it simplifies the point cloud, giving it an advantages from the former cited.

3 METHODOLOGY

3.1 SOFTWARE

Point Cloud manipulation requires specialized software even if the data structure is pretty simple. For this work all the implementations were done in MatLab, MeshLab and Point Cloud Library (PCL) [39], a C++ code for point cloud manipulation.

3.2 PROCEDURE

The test method occurred on a simple loop of action. Read the data from any source, filter it on MeshLab and compare the quality results with others. First the data must be on the Stanford Point Format called Polygon File Format (PLY). For that a simple MatLab code for format conversion is used to transform the data.

Then the file is imported to MeshLab by going into File > Import Mesh Fig 3.1. Filtering can start the point cloud shown on the monitor. Most of the filtering techniques require extra information to work, like the point normals. For that on MeshLab go to Filters>Point Set>Compute Point Normals. The parameter to change is the Filter Scale for MLS and Number of Samples for Point Cloud Simplification.

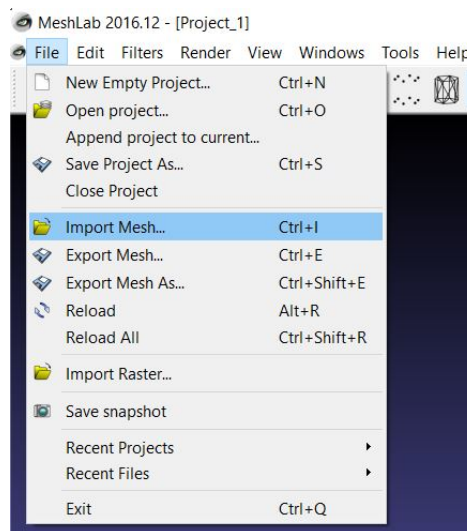


Figure 3.1 – Import Point Cloud in MeshLab

When scanning objects an artifact appears. The artifact is the strange toroid feature that appears on top of scanned objects Fig 3.2. It is assumed that it happens because the

sensor measure faraway points and close points in its measurement window. When the sensor average those samples they get sparser. This phenomenon is yet to be investigated. For better comparison the scanned data is split in two, one with the top artifact and another where it is manually removed. The sensor aperture measures far and close points at the same time. To remove points on the top right corner of the action menu click select vertexes and then after selecting the points considered to be outliers click delete vertexes.

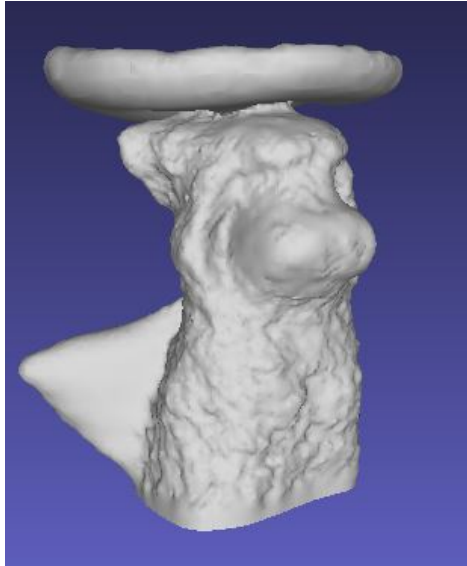


Figure 3.2 – Import Point Cloud in MeshLab

The filtering occurs by going on the Filter tab again an selection Point Set > either MLS projection or Point Cloud Simplification 3.3 the other filter are in Filters> Smoothing, Fairing and Deformation> Laplacian Smooth or Taubain Smooth. The changed parameters are number of steps.

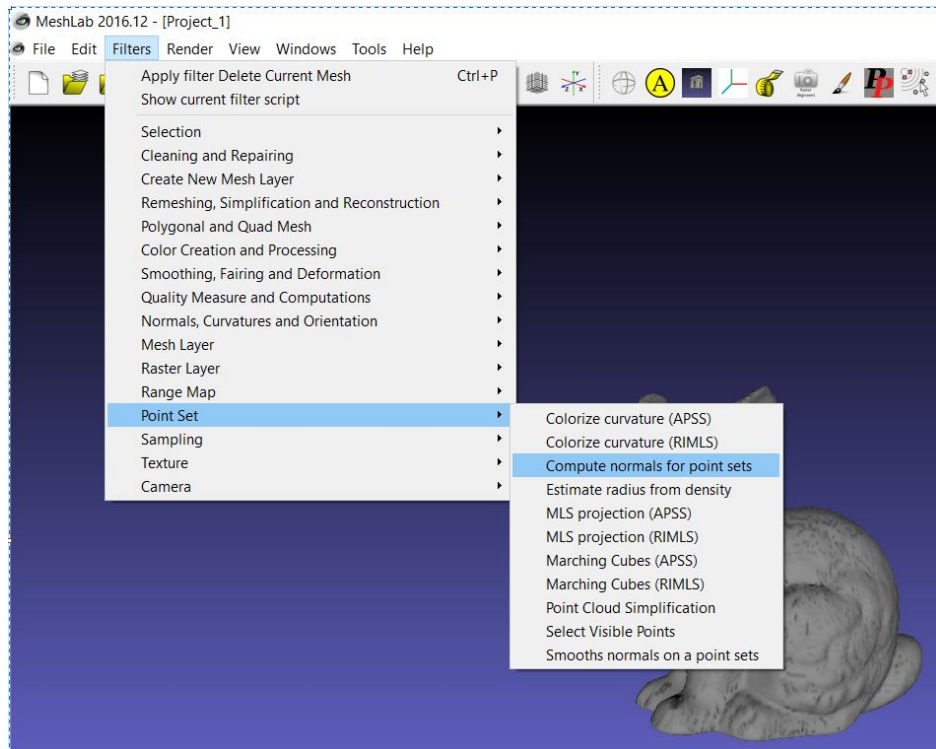


Figure 3.3 – Normal Estimation, MLS and Downsampling options

Noisy synthetic data is generated by importing the original mesh and going to Filters > Smoothing, Fairing and Deformation > Random Vertex Displacement. This action will add gaussian noise to the data.

Lastly the point cloud is turned into a surface with the Poisson method. To do so go in Filters > Remeshing, Simplification and Reconstruction > Screened Poisson Surface Reconstruction. The default values were used to generate the surfaces.

The test will be made using synthetic point clouds and point clouds sampled from the 3D scanner. They will be compared visually for the quality of the reconstruction. The filters applied to them will be using the values of Table 3.1 using the process shown in Fig 3.4.

Method	Parameters	Values
MLS	Filter Scale	4
	Projection Accuracy	10
	Projection Max Iterations	0.0001
Laplace	Smoothing Steps	15
Taubain	Lambda	3
	mu	1
	Smoothing steps	0.53
Downsampling	Number of Samples	10
		1000

Table 3.1 – Table with the filters Parameters

3.3 FLOWCHART

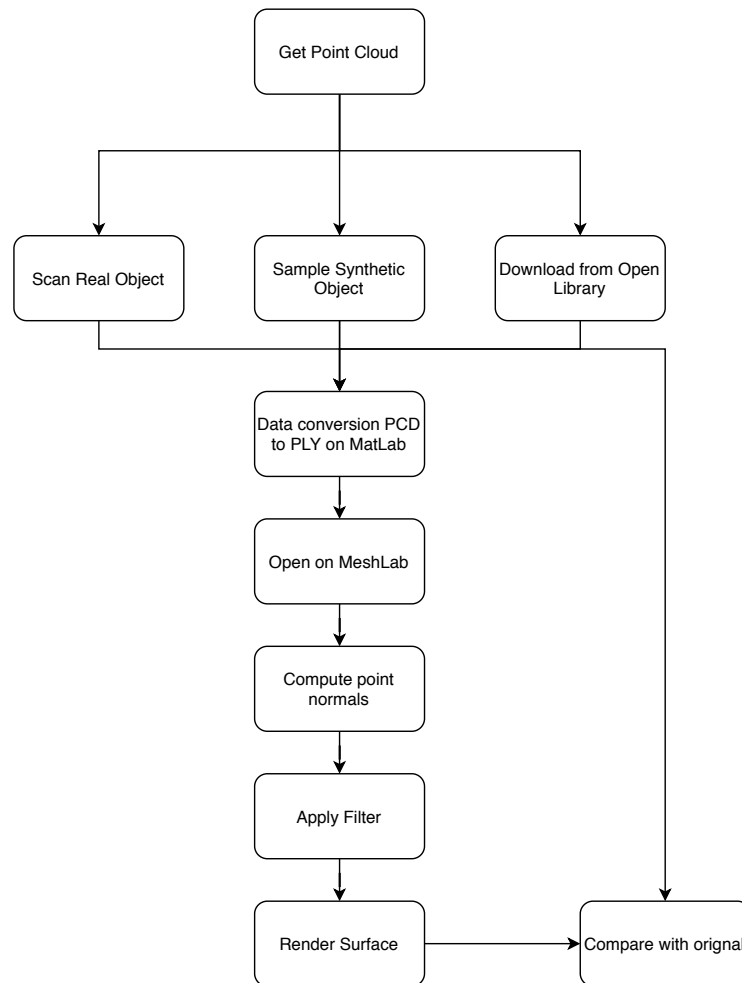


Figure 3.4 – Procedure Flowchart

4 RESULTS

In order to compare the visual result of both of them. The results are divided in synthetic and sampled.

4.1 SYNTHETIC RESULTS

The filter algorithms were tested in a synthetic point cloud in the shape of a bunny, provided by the University of Stanford [40]. Shown in Fig 4.1.

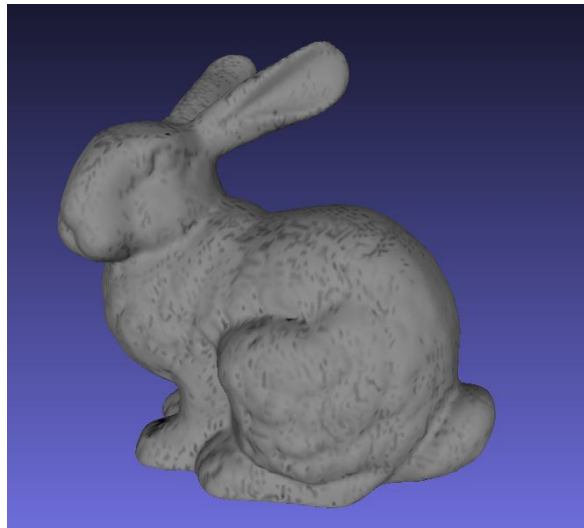


Figure 4.1 – Original synthetic point cloud

Then Apply gaussian noise to the point cloud as in Fig 4.2.

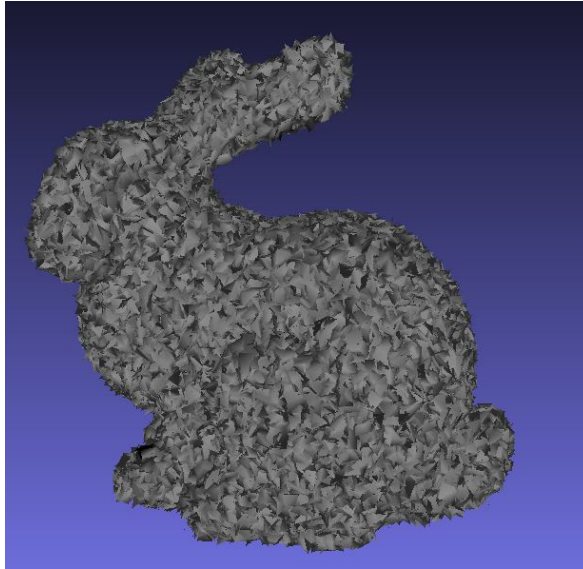


Figure 4.2 – Noisy synthetic point cloud

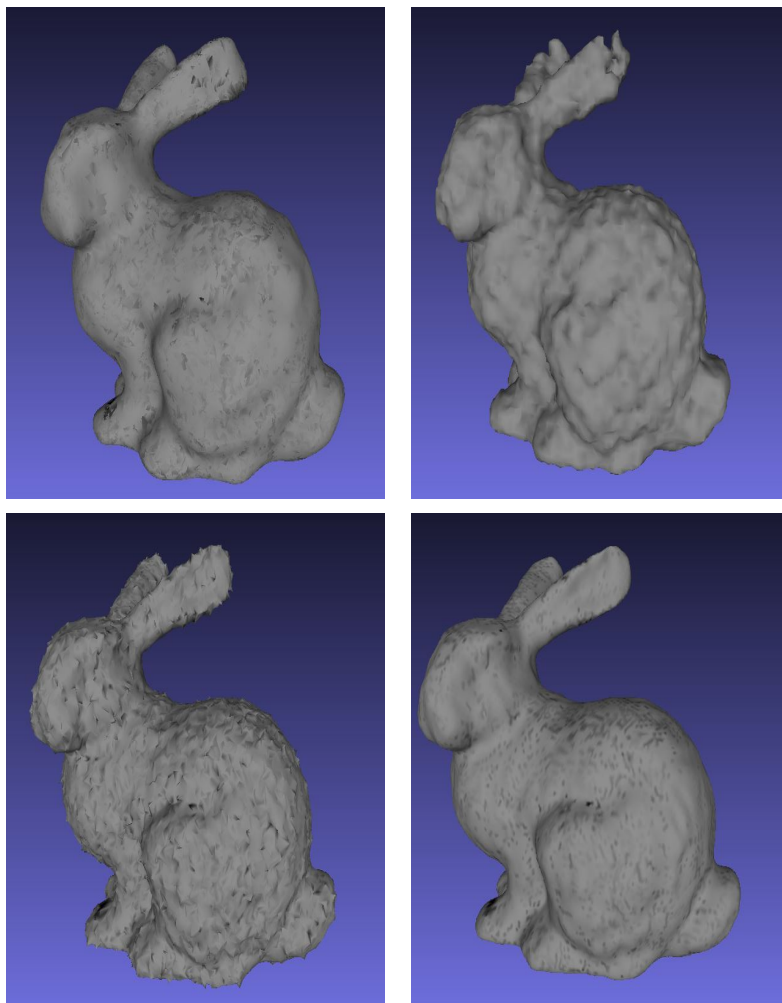


Figure 4.3 – Top left is the MLS filtered, Top right is the downsampling filtered, bottom left is Laplacian operator and bottom right is Taubain operator

As shown in Fig 4.3 all filter recover the original shape of the object, however the down-sampling and the Laplacian operator achieves poorer results, which is expected from simpler methods. The surface is very rough. Great distortions can be noted in the bunny ears at the downsampling result.

The MLS and Taubain methods recover better hulls, the Taubain operator achieves better reconstruction of sharp features, the bunny snout and eye region show more detail. This also match the expectation, since the Taubain operator is a more refined technique.

4.2 SAMPLED RESULTS

For the sampled results a doll was sampled. As shown in Fig 4.4. The data acquired directly from the sensor is very noisy, what can clearly be seen in Fig 4.5. Manually removing the artifacts generated by the sensor incapability to acquire sharp edges from small objects gives more form to the object.



Figure 4.4 – Original object scanned

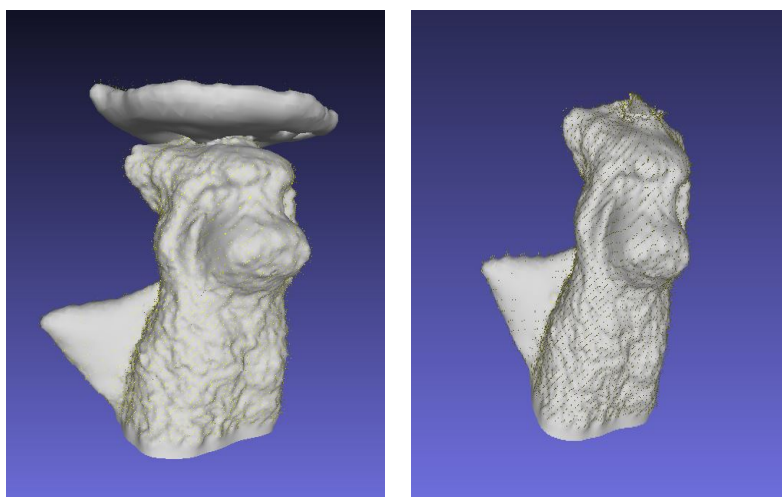


Figure 4.5 – Raw data rendered at the left and the removal of the artifact

4.2.1 Projection based method

Applying the MLS algorithm shows how the artifact effect the processing of the point cloud. Since MLS is a smoothing procedure in tends to merge surfaces in order to achieve less noisy shallows. As shown in Fig 4.6.

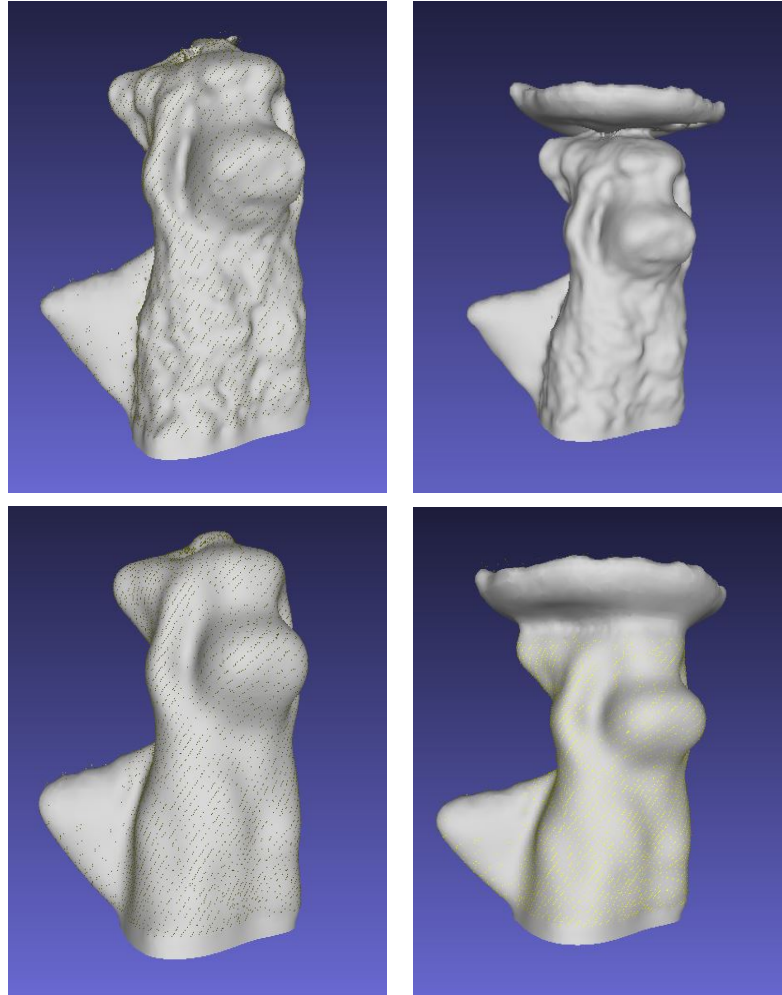


Figure 4.6 – First row the MLS with size 4 is applied, second row the MLS with size 10

The size of the MLS algorithm changes how much noise will be removed. Scaling up the filter create a smoother surface. However it also round sharp features. Applying a small kernel leaves the surface still noisy. The bigger one smoothes features together.

Note the artifact has a lot of weight in filtering, it basically merges itself in some sort of a crown, damaging the reconstruction. As stated before in the review the MLS filter causes shrinkage of the point cloud. However the factor used above does not lead to a good visual cue. Fig 4.7 shows the point cloud with the filter at size 50. The whole object turns into cylinder.

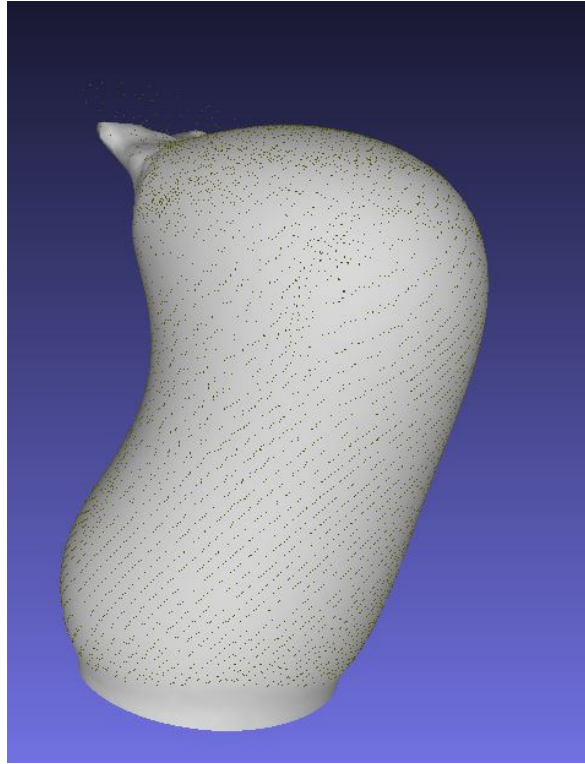


Figure 4.7 – Point cloud after the MLS with size 50

4.2.2 Signal processing method

Results of applying the Laplacian operator through the point cloud Fig 4.8. The Laplacian operator smooth surfaces indiscriminately. It is a fairly simple matrix operation, reduces small amplitude noise leaving rougher surfaces noisy.

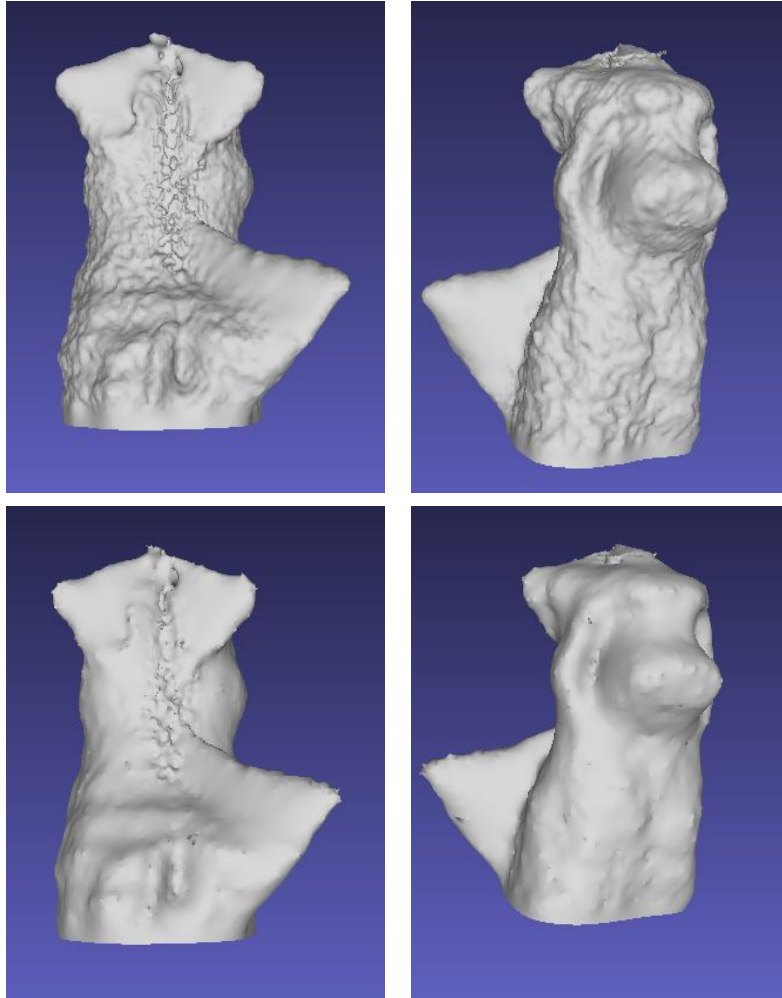


Figure 4.8 – Top row raw pint data and bottom row the Laplacian operator

As displayed the Laplacian operator smooth most of the surface eliminating the acquisition noise, yet at the back of the object there is still rough regions.

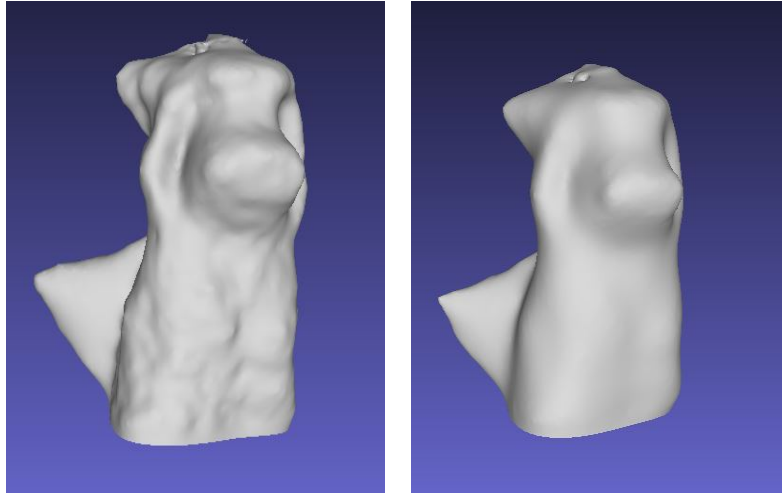


Figure 4.9 – First figure Taubain operator with 10 steps, second figure Taubain with 50 steps

Taubain operator shows a smoother surface and the sharp features get smoothed more than MLS. However since the shrinkage parameter is controllable, the point cloud is not deformed as much. Proving that Taubain operator is the most adequate method for this stage of the 3D scanner.

4.3 SURFACE CHARACTERISTICS WEIGHT

The sensor utilizes light to measure objects. Therefore the objects color, reflexivity and refractibility weights on the measurement. Darker colors makes the object appear more distant while bright colors make the object appear closer. This happens because darker colors absorbs more light frequency and therefore the signal reflected on them is weaker. A weaker signal may appear to be a distant signal for the sensor. The opacity of the object seems also more distant, since the light goes trough it and reflects behind the surface. The signal pervieved by the receiver is measured more distant. The Fig 4.10 shows the effect where covering the red cheek of the Pikachu toy makes the measurement appear closer. However the black of the eyes still creates sags on the scanned reconstruction.

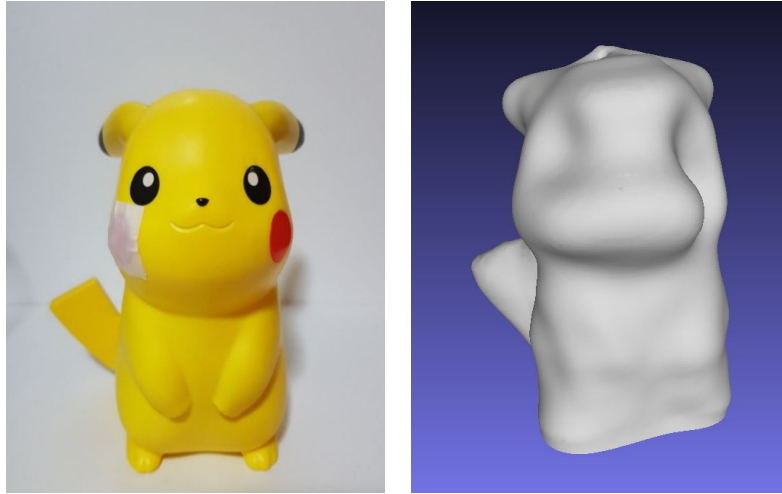


Figure 4.10 – Scan with the occluded translucent part

5 CONCLUSION

Point Cloud filtering definitely helps at the reconstruction of 3D scanned objects. Removing noise and retrieving sharper features is possible. However it is clearly shown that for this setup the quality of the scan is not matched to the desired objective of reconstruction of historical artifacts. Simpler objects that are less textured may show better results. Since the main objective is a low-cost proposition of 3D scanning a better treatment for the sensor modeling should help the results.

One thing that is well clear for all the applications reviewed in this work is the specificity of every one of them. Every solution is uniquely attached to a single problem, i.e. connected to a specific setup or sensor. For better performance in reconstruction the filtering technique must keep evolving together with the 3D scanner evolution. Considering this is the first iteration on the software level of the project and many more to come, filtering shows great potential for object precise reconstruction.

The statistical techniques when applied with the projection techniques show good results in denoising and reconstruction of complex objects. These techniques can treat edges and sharp features well. However they are time costly. Signal processing techniques are simpler and can smooth small amplitude noise. These are better suited for denoising objects with simpler geometrical configuration and precise sampling. The downsampling technique is not appropriate for this project. The number of samples are too small for denoising by simplification.

5.1 FUTURE WORK

- Modeling of the sensor activity during the scanning, so simpler noise and outliers canceling methods can be performed. For example the application of Kalman filter for the distance measurement. This change may help canceling the appearance of artifacts on edges. It is more robust than moving average algorithm. The later is heavily influenced by outliers.
- Addition of more sensor so multi signal filtering and data fusion can be applied. The addition to a secondary sensor that capture the same scene creates redundancy. More sensor leads to uncertainty reduction and better samples. The second sensor can be on top of the rotatory platform. In this configuration it wont have much correlation with the main sensor. However by measuring the top of objects uprightly may reduce the artifact appeared on top of them.

- Study of color effect on scanned data, the results show a huge bias of the sensor when it comes to scanning different colored parts of the same surface. Scanning known objects. For example cylinders made with known height and radius. Then coloring them with different colors and patterns. After scanning the distortion can be actually measured.

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