BACHELOR THESIS

MODEL FOR OBTAINING THE SET OF ALL NASH EQUILIBRIUM SOLUTIONS IN A PORTFOLIO MANAGEMENT OF THE ENERGY SECTOR

By,
Bárbara Caldeira Macedo

Brasília, June 19th, 2019

UNIVERSIDADE DE BRASILIA

FACULDADE DE TECNOLOGIA

DEPARTAMENTO DE ENGENHARIA DE PRODUÇÃO

UNIVERSIDADE DE BRASÍLIA
Faculdade de Tecnologia
Departamento de Engenharia de Produção

PROJETO DE GRADUAÇÃO

MODEL FOR OBTAINING THE SET OF ALL NASH EQUILIBRIUM SOLUTIONS IN A PORTFOLIO MANAGEMENT OF THE ENERGY SECTOR

Por,
Bárbara Caldeira Macedo

Relatório submetido como requisito parcial para obtenção do grau de Engenheiro de Produção

Banca Examinadora

Prof. Ph.D Reinaldo Crispiniano Garcia, UnB/EPR (Orientador) ____________________________________

Prof. Dr. Victor Rafael Rezende Celestino, UnB/ADM ____________________________________

Brasília, 19 de junho de 2019

ABSTRACT

The electricity market has experienced a major worldwide deregulation process over the last 40 years. In the deregulated electricity markets the involved parties — the buyers and the producers — negotiate electricity deliveries through bilateral contracts or in the spot market. This deregulation made the energy sector more competitive and efficient but at the same time subject to the risks of the spot prices volatility. The contract parties engage in optimal trading strategies in such a way to obtain the highest possible profits.

Many studies of the energy sector propose a game theory approach to tackle this conflict. This work applies the Nash bargaining solution to investigate the optimization problem for contract scheduling. A procedure to find all Nash equilibria is developed by generating "holes" added as linear constraints to the feasibility region. The mathematical model is applied in the MATLAB software.

Other studies present the Raiffa-Kalai-Smorondinski (RKS) approach as an alternative to Nash bargaining to find a compromise solution. After obtaining the set of all Nash solutions, a comparison was made between the number of solutions found in both equilibria approaches. It is shown that the amount of solutions with the Nash equilibrium approach is the same as with the RKS approach.

RESUMO

O mercado de eletricidade passou por um grande processo de desregulamentação mundial nos últimos 40 anos. No mercado de eletricidade desregulamentado, as partes envolvidas - os compradores e os produtores - negociam as entregas de eletricidade por meio de contratos bilaterais ou no mercado spot. Essa desregulamentação tornou o setor energético mais competitivo e eficiente, mas, ao mesmo tempo, sujeito aos riscos da volatilidade dos preços spot. As partes contratuais envolvem-se em estratégias de negociação ótimas de forma a obter os maiores lucros possíveis.

Muitos estudos do setor energético propõem uma abordagem da teoria dos jogos para enfrentar esse conflito. Este trabalho aplica a solução de Nash para investigar o problema de otimização da negociação de contratos. Um procedimento para encontrar todos os equilíbrios de Nash é aplicado gerando "buracos" adicionados como restrições lineares à região viável. O modelo matemático é aplicado no software MATLAB.

Outros estudos apresentam a abordagem Raiffa-Kalai-Smorondinski (RKS) como uma alternativa à negociação de Nash para encontrar uma solução compromissada. Depois de obter o conjunto de todas as soluções da Nash, foi feita uma comparação entre o número de soluções encontradas em ambas as abordagens de equilíbrio. É apresentado que a quantidade de soluções com a abordagem de equilíbrio de Nash é a mesma que com a abordagem RKS.

TABLE OF CONTENTS

LIST OF SYMBOLS

Variables

x_k^t	Amount of electricity delivered under the BC and sold to end consumers at interval t, MWh
x_s^t	Amount of electricity bought by ESC in the spot Market at interval t, MWh
x_{ss}^t	Amount of electricity received by ESC under the BC and sold in the spot market at interval t, MWh
x_c^t	Amount of electricity produced by generation company (GC) and delivered under the BC at interval t, MWh
x_{gss}^t	Amount of electricity produced by GC and sold in the spot market at interval t, MWh
x_{gs}^t	Amount of electricity bought by GC in the spot market and delivered under the BC at interval t, MWh
x_g^t	Output of GC at interval t, MWh
x_d^t	Amount of electricity sold to end consumers at time interval t, MWh
R_1, R_2	Revenues obtained by contract parties from participation in the spot market, managing BC and supplying electricity to em costumers, $
S_1, S_2	Profits gained by contract parties from participation in the spot market, managing BC and supplying electricity to em costumers, $
$\Delta S_1, \Delta S_2$	Relative concessions of the contract parties applying the compromisse
J	Contract price equal to the total agreed cost of electricity delivered under the BC, $

Random variable

$\tilde{p}_s^t$	Forecasted electricity price in the spot market at time interval t, \$/MWh

Constants

p_d^t	Electricity price for end consumers in the retail market at time interval t, \$/MWh
V	Total amount of electricity received by ESC under the BC during the contract period, MWh
x_{min}^t, x_{max}^t	Limits for contract deliveries at interval t, MWh
$x_{g\,min}^t$, $x_{g\,max}^t$	Limits for electricity production of GC at interval t, MWh

Function

$C^t(x_g^t)$	Production cost function of GC at time interval t, \$

Number

N	Number of time intervals in the contract period

LIST OF FIGURES

LIST OF TABLES

1. INTRODUCTION

Electricity is one of the leading energy sources, second only to fossil fuels (U.S. ENERGY ADMINISTRATION, 2013 cited BARBOSA, 2017). By 2040, world electricity consumption is predicted to be 34.5 trillion kWh - 57% higher than in the year 2015 (IEA, 2017). Other studies show even higher expectations - growth of 30% by 2035 - or even a demand growth of 48% by the year 2040 (BRITISH PETROLEUM, 2017; US ENERGY INFORMATION ADMINISTRATION, 2016). The main determinants of energy growth can be explained by economic growth which strongly influences energy consumption, and also by the emergence of electric cars, whose projection of energy demand has grown considerably (IEA, 2017).

In the late 1980s, the energy market experienced a major deregulation process throughout worldwide. This deregulation process which started in the 1980s, aims to open the energy market in such a way that it becomes a competitive, efficient, and reliable market with low tariffs to end consumers.

In a deregulated market, energy is traded as a commodity as is iron, oil or natural gas (BARBOSA,2017). Market agents (generators, distributors, retailers, and end consumers) have the autonomy to exchange energy with any other agent, establishing the volume, price, and terms of supply. In this market, the energy can be negotiated in two ways: a) the "day ahead" market, e.g., spot market, which refers to the transactions carried out in the short term; and b) the future market, where the operations are made through bilateral contracts (BC) for medium and long term. The bilateral contracts secure delivery of a certain amount of electricity at agreed prices (PALAMARCHUK, 2012). On the other hand, the spot market trades present highly volatile prices since energy cannot be stored.

The deregulated market allows actors to engage in optimal trading strategies through bilateral contracts or spot market. Electricity received under BC may be resold in the spot market and electricity bought in the spot market may be delivered to fulfill bilateral contracts obligations

(PALAMARCHUK, 2012). These two markets provide electricity supply to society and industry at a fair price, since it reduces losses of inefficiency market.

Following the deregulation of the world's energy markets, which began in the mid-1980s, several studies began to be conducted in order to deepen the knowledge on price prediction (ERNI, 2012; WERON, 2014; URYASEV, 2000; BERTSIMAS et al, 2004; POUSINHO et al, 2013; GARCIA, 2005). These studies intend to maximize the profits of the involved parties in the energy market with techniques of forecasting of price and demand. Different methods are used to address the problem, such as the use of game theory, which seeks to obtain a solution which is located at the Nash equilibrium point in the BC negotiation.

Negotiation techniques pose a major challenge to optimize the agents' revenues given the buying and selling decision alternatives. In order to tackle this problem, Cunha (2017), Toledo (2017) and Barbosa (2017) present two types of equilibrium solutions for an electricity scheduling problem proposed by Palamarchuk (2010). The first approach concerns of an optimal contract condition according to the Nash bargain solution. The second proposed solution uses the Raiffa-Kalais-Smorodinsky (RKS) equilibrium as an alternative to the Nash bargain solution. In this case, the RKS solution resulted in a smaller concession value, that is, higher profits compared to the Nash solution.

Furthermore, several simulations were carried out with different input data and the authors were successful in demonstrating the existence of other optimal solutions to the problem when applying the RKS approach. The necessity arises to develop an algorithm that finds all the set of optimal solutions for this research problem. A mathematical model was formulated and found all the solutions with the RKS approach (MONTEIRO, 2018). The present work aims to provide a model to find, then, the set of Nash bargain solutions. The algorithm is formulated and is used to find all the Nash bargain solutions. Lastly, it compares the set of solutions found with the RKS equilibrium in the work of Monteiro (2018). It is also important to compare the number of solutions that each equilibrium approach provides due to the flexibility it gives to the agents. In case that the number of solutions is bigger in one approach, that means that it gives more options for the decision makers in the contract scheduling.

1.1 OBJECTIVES

This project aims to identify the set of all optimal Nash equilibrium solutions in a dynamic energy bilateral contract scheduling. In order to do so, a mathematical model will be developed based on the concepts of operations research and game theory.

1.2 MAIN OBJECTIVE

The main objective of the current project can be explained as:

"The development of a mathematical model for a dynamic energy bilateral contract scheduling with the purpose of identifying the set of all optimal Nash equilibrium solutions to maximize the profits for the involved parties."

1.3 SPECIFIC OBJECTIVES

The main objective can be subdivided into four separate specific objectives:

- Analyze the Nash bargain problem of the energy sector;
- Investigate the method for obtaining the set of all optimal solutions;
- Design and implement the mathematical model that provides the set of all optimal solutions;
- Compare the results obtained from the Nash bargain solution with those of the RKS solution.

1.4 PROJECT STRUCTURE

The structure of this work is organized as follows:

Chapter 1 - INTRODUCTION, describes the context, motivation, and the objective of this project.

Chapter 2 - METHODOLOGY, presents the research strategy used in this project.

Chapter 3 - LITERATURE REVIEW, introduces the reader to the field of electricity markets, the bilateral contract negotiations, and the Spot market. Furthermore, this chapter presents the theoretical background for the development of the mathematical model.

Chapter 4 - THE DEVELOPMENT OF THE MODEL, presents the problem designed by Palamarchuk and a numerical example to illustrate the problem. Subsequently, it is proposed a model to find the set of Nash bargaining solutions for the problem presented by Palamarchuk.

Chapter 5 - RESULTS. This chapter provides the results of the set of all Nash equilibria obtained with the algorithm applied in the MATLAB software. Later, we present a comparison with the results obtained with the Raiffa-Kalais-Smorodinsky approach.

Chapter 6 - CONCLUSION AND FUTURE STUDIES. In this chapter, a reflection is given on the work done and topics for future work are proposed.

2. METHODOLOGY

In order to classify this work it is necessary to introduce some important concepts of research classification. This chapter will describe different research classifications and will present the research approach used in the project.

2.1 RESEARCH CLASSIFICATION

Scientific research is a systematic, controlled, and critical reflexive procedure that allows the discovery of new facts or data, new relationships or laws, in any field of knowledge (ANDER-EGG, 1978). It is then necessary to classify the research according to some criteria. Scientific research can be classified according to the research method, objectives, procedures, and nature (SILVEIRA; CÓRDOVA, 2009).

From the point of view of the method, the research can be quantitative, qualitative, or a mixed approach (CRESWELL, 2010). In the quantitative one, the study focuses on objectivity. The quantitative method deals with numbers and anything that is measurable to explain, predict, and control a phenomenon (LEEDY, 1993). On the other hand, the qualitative method does not require the use of statistic and mathematical tools. In this method, the researcher is focused on the aspects that can not be quantified to understand the dynamics of social relations (SILVEIRA; CÓRDOVA, 2009). Lastly, the mixed method focuses on collecting, analyzing, and mixing both quantitative and qualitative data in a single study or a series of studies (CRESWELL, 2010). The central premise of the mixed method is the use of quantitative and qualitative approaches combined, providing a better understanding of the research problems than either approach alone (CRESWELL, 2010).

According to the objectives, scientific research can be classified as exploratory, descriptive, or explanatory (GIL, 2008). Exploratory research aims to provide greater familiarity with the problem. It involves a bibliographical survey, interviews with people with experience with the researched issue, and the analysis of an example that gives a better understanding of the problem (GIL, 2008). The majority of the research done for academic purposes, at least in some level, has an exploratory approach (GIL, 2008). The descriptive research requires a series of information about what the researcher wants to investigate. This type of study aims to describe the facts and phenomena of a certain reality (TRIVIÑOS, 1987). Examples of such projects are: case studies and documentary analysis. Explanatory study, on the other hand, aims to identify the factors that determine or contribute to occurrence of a phenomena (GIL, 2008). Explanatory research may be the continuation of another descriptive one, since the identification of the factor that determines a certain phenomenon requires that it be sufficiently described and detailed.

In terms of the procedures, there are seven types of research as follows (GIL, 2008):

Table 1 - Types of research

Bibliographic	This type of research is based on material that is already published, and consists mainly of books and scientific articles.
Documentary	Similar to the bibliographical one. The difference lies in the nature of the sources, since this procedural form is made of materials that have not yet received an analytical treatment.
Experimental	When a study object is determined, the variables that can influence the study are selected and the forms of control and the observation of the effects that the variable produces on the object are defined.
Survey	It is the direct interrogation of people whose behavior the researcher wishes to know about. It is necessary to request information from a significant group of people about the problem studied and then, through quantitative analysis, obtain the conclusions corresponding to the collected data.
Field Study	It involves an in-depth and exhaustive study of a specific reality. It is performed through the direct observation of the activities of the studied group and through interviews with sources to capture the explanations and interpretations of the field.
Case Study	Consists of an in-depth and exhaustive study of one or a few objects, in a way that allows for ample and

	detailed knowledge to be collected.
Action Research	An empirically based type of research that is conceived and carried out in close association with an action or with the resolution of a collective problem in which researchers and participants of the situation or problem are involved in a cooperative or participatory manner.

Finally, according to nature, the research can be classified as basic or applied. Basic research aims to generate new knowledge that is useful for advances in science without the practical application. Applied research, however, aims to generate knowledge for practical application and is targeted to the solution of specific problems.

After evaluating the research definitions, it is possible to conclude that this work follows a quantitative approach, has a descriptive objective, is based on bibliographic procedures, and is classified as applied research.

3.LITERATURE REVIEW

3.1. ELECTRICITY MARKET SYSTEM

For nearly a century, the electricity market system in all countries has been thought of a "natural" monopoly industry (CHAO; HUNTINGTON, 1998). Such a scenario required reliance on public or private monopoly subject to the government regulation to achieve efficient production. However, over the last decades, the electricity market system has been forced to transform the way of operation in many countries. The main argument of the deregulation of the market system is the inefficiency of regulation (STOFT, 2002).

The reasons of changing from vertically integrated mechanisms to open markets have differed over regions and countries (ABHYANKAR, KHAPARDE, 2013): In developing countries, the main problems have been a high demand growth combined with inefficient system management and unreasonable tariff policies. Those problems have affected the accessibility of financial resources to improve generation and transmission capacities. For developed countries, on the other hand, the motivation has been to provide electricity at lower prices and offer them a greater choice in purchasing economic energy.

A deregulated market promotes an extremely competitive environment. In this model, there is certain autonomy regarding the amount of generation and consumption negotiated as well as their respective prices: Consumers and suppliers arrange trades independently. Through competition in liberalized markets, incentives are created to drive for the more efficient operation of electricity systems (IEA, 2005). These include labor saving techniques, more efficient repairs, less costly construction on new plants, and wiser investment choices in terms of timing, sizing, siting and choice of technology (STOFT, 2002; IEA, 2005).

Contract parties schedule electricity deliveries over a contract period to obtain the highest possible profit. One of the methods to achieve the maximum profit is contract portfolio management. This method seeks to obtain the diversification of the agreed contracts (DE LLANO-PAZ ET AL, 2017; cited BARBOSA, 2017). Each contract is subject to different constraints and associated risks. Thus, the agents of the energy market usually act both on the spot market and bilateral contracts (future market). In the next sections, we will describe each one of the contracts for energy transactions. To do that, first, a definition of the electricity market agents will be provided.

3.1.1. ACTORS

The electricity market is composed of the following actors (KIRSCHEN, 2004):

Table 2 - Actors of the electricity market

ACTORS	
Generating Companies	Produce and sell electric energy. A generating company can own a single plant or a portfolio of plants with various technologies. There are several technologies for generating energy, the most common being coal and peat, natural gas, hydroelectric, nuclear, and oil (OECD, 2016). The producers can sell both to the electric market and the end consumers.
Distribution Companies	Own and operate distribution networks. In a fully deregulated condition, the sale of electricity to consumers is decoupled from an operation, maintenance and development of the distribution network. Retailers then compete to perform this energy sale activity. One of these retailers may be a subsidiary of the local distribution company.
Retailers	Buy electrical energy on the wholesale market and resell it. They resell the energy to consumers that either do not want to be part of the wholesale market or are not allowed to participate. The retailers can buy electricity throw stock trading, futures trading, or bilateral contracts.
Market Operators	Usually runs the computer systems that matches the bids and offers that buyers and sellers of electricity have submitted. It also deals with the settlement of the accepted bids and offers. That is it forwards payments from buyers to sellers following the delivery of electricity.
Independent System Operator	Its main responsibility is to maintain the security of the energy system. The term independent is due to the necessity to have an

	impartial agent in a competitive environment. In such manner, the system is operated so the system does not favor or penalize any of the involved parties.
Transmission Companies	Own transmission assets, for example, lines, cables, transformers, and reactive compensation devices. Transmission companies operate the equipment following the directions of the independent system operator (ISO). They are often subsidiaries of companies that also own generating plants.
Regulator	It is the government institution that is responsible to ensure the fair and efficient operation of the energy sector. It establishes and approves the rules of the energy market. Moreover, it deals with investigation of cases of abuse of market power. The regulator also sets the prices for products and services which are provided by monopolies.
Small Consumers	Buy electricity from a retailer and lease a connection to the power system from their proximate distribution company. Their participation typically amounts no more than choosing one retailer among others when the option is available.
Large Consumers	Usually, have an active role in the energy market because they buy their electrical energy directly through the market. The biggest of those consumers are sometimes directly connected to the transmission system.

In this study, the focus will be on a generation company (GC), namely the supplier of energy, and on an electricity supply company (ESC), the buyer of electricity, that sells it to the end consumers.

3.1.2. SPOT MARKET

Typically, electricity markets are divided into spot or future transactions. This distinction is based on the time at which transactions are to be completed (BURNS, 1979). Spot markets consist of transactions which are completed immediately. It is also called the real-time market or the day-ahead market. In the spot market, the seller delivers the goods right away, and the buyer pays for them "on the spot."

By the very nature of electricity, it is not possible to store. Consequently, the prices in the spot market are very uncertain. Given that the spot market consists of instant transactions, the actors cannot anticipate the cost of contract delivery, accurately (Jamshidi, 2018). The prices in the spot market are established in an auction-based system, where traders hand in prices and volumes for production or consumption for given hours the next day (BENTH, SCHMECK, 2014). Based on these bids, the market regulator generates the demand and supply curves.

This market is commonly used combined with bilateral contracts. For example, generating companies (GC) can use it as a source to purchase energy so they can meet a current promise. The reasons to do that can be inadequate production planning, insufficient energy generation, or the simple fact that the price in the spot market at a particular period compensates for the decision not to produce a particular amount of energy to be delivered through a bilateral contract.

3.1.3. BILATERAL CONTRACTS

In this setting, a buyer and a seller agree on a certain amount of energy to be transferred at a fixed price. The contracts can cover periods ranging from weeks until years at a pre-established price. Therefore, they provide greater stability in the demand and supply of electricity.

Bilateral contracts are often used to reduce the spot price risks of its uncertainty and volatility. High demand, for example, culminates in a large increase in the electricity price. Since the spot market prices, due to their uncertainty, may be higher or lower than predicted, bilateral contracts are an essential tool to protect and mitigate risks.

This type of contract is negotiated based on expected long-run price averages. Although the contract can be used as a means to avoid risk, the spot bidding may also provide the possibility of higher profits to the actors. High prices in the spot market are favorable for the generation company (GC) to increase sales in this market and reduce deliveries under BC. Simultaneously, it is profitable for the electricity supply company (ESC) to decrease purchase from the spot market and increase contract deliveries. Therefore, the involved parties need to manage the combination of bilateral contracts and spot market transactions searching for profit maximization (Xia et al, 2019).

3.2.OPERATIONS RESEARCH

Operations research (OR) is a scientific approach to decisions (KAUFMANN, 1963). The term gained significant recognition in World War II. At that time, there was a great need to allocate scarce resources in various military operations to achieve each operation in an effective manner (HILLIER, 2004). For this reason, the British and U.S. army invited a group of scientists to apply a scientific approach in order to solve strategic and tactical problems (HILLIER, 2004). It was used to deploy radars, manage convoys, prioritize bombing missions, and control anti-submarine operations (WANG, 2017).

After World War II, the success of operations research aroused great interest in applying the methods outside the military sector. Since then, the business world has been investing heavily to put the operations research studies together with the industry's needs. Typical examples of OR techniques in the business sector are routing, logistics, investment portfolio definition, planning forecasts, scheduling, project analysis, queueing optimization, along with others. The operations research description nowadays is commonly accepted as a scientific approach for better decisions in a complex and uncertain environment (WAGNER, 1975).

While the statistical method is a very ancient means of treating information, the analytical techniques are recent developments (KAUFMANN, 1963). Operations research requires the use of models, which are mathematical representations of the actual systems. An optimization model seeks to find values of the decision variables that optimize (maximize or minimize) one or more objective functions among a set of all values for the decision variables that satisfy the given constraints.

Even though Operations research is based on quantitative techniques, it does not mean that practical OR studies are primarily numerical exercises. The mathematical analysis, in reality, is a rather small part of the total effort (HILLIER, 2004). Operations research applies a scientific research methodology that can be summarized in the following phases (HILLIER, 2004):

1. Define the problem of interest and gather relevant data;
2. Formulate a mathematical model to represent the problem;
3. Develop a computer-based procedure for deriving solutions to the problem from the model;
4. Test the model and refine it as needed;
5. Prepare for the ongoing application of the model as prescribed by management;
6. Implement.

This work has an operations research approach until step 4 - given that the test results of the proposed mathematical model will be presented - so it can later be improved and implemented.

3.4 GAME THEORY

Game theory is the discipline that provides mathematical techniques for analyzing the situations in which two or more individuals (called agents or players) make decisions that influence one another's welfare. Each agent's decision is ordered among multiple alternatives captured in an objective function for that player. The objective function either tries to maximize (in which case the objective functions is a utility function or benefit function) or minimize (in which case we refer to the objective function as a cost function or a loss function) (BASAR, 2010). It can be used to study parlor games, political negotiation, and economic behavior (VARIAN, 2006). One of the most important concepts of game theory is the Nash equilibrium.

3.4.1. NASH EQUILIBRIUM

Nash equilibrium is a fundamental concept in game theory and the most widely used method of predicting the outcome of strategic interaction. It was named after the mathematician John Forbes Nash Jr., who presented, in 1950, a solution of a non-cooperative game.

The concept of Nash equilibrium relies heavily on individual rationality (PINDYCK, RUBINFELD, 2013). The theory is based on a set of strategies (or actions) such that each agent is doing the best it can given the actions of its opponents. Each agent's choice of strategy depends not only on its own rationality but also on that of its opponent (PINDYCK, RUBINFELD, 2013). Thus, a Nash equilibrium requires that the agents be right about the assumptions of the strategies chosen by the other agent (RESENDE, 2019)

Stated simply, the Nash equilibrium can be defined as the point of a system that involves different agent's interaction in which no agent can improve her payoff with a single move of strategy if the other agent's strategy remains the same.

Mathematically (NARAHARI, 2012):

Given a strategic form game $\Gamma = \langle N,\ (S_i),\ (u_i) \rangle$, the strategy profile $s^* = \left(s_1^*, s_2^*, \ldots, s_n^*\right)$ is said to be a pure strategy Nash equilibrium of Γ if,

$$u_i\left(s_1^*,\ s_{-i}^*\right) \geq u_i\,(s_1,\ s_{-i}),\quad \forall s_i \in S_i,\ \forall i = 1, 2, \ldots, n \qquad \textbf{(1)}$$

That is, each agent’s Nash equilibrium strategy is the best response to the Nash equilibrium strategies of the other players.

Given a game $\Gamma = \langle N,\ (S_i),\ (u_i) \rangle$, the best response correspondence for player i is the mapping $B_i : S_{-i} \rightarrow 2^{S_i}$ defined by

$$B_i\,(s_{-i}) = \left\{s_i \in S_i : u_i\,(s_i, s_{-i}) \geq u_i\left(s_i', s_{-i}\right)\ \ \forall s_i' \in S_i\right\} \qquad \textbf{(2)}$$

That is, given a profile s_{-i} of strategies of the other players, $B_i(s_{-i})$ gives the set of all best response strategies of player *i*.

It can be seen that the strategy profile $\left(s_1^*, \ldots, s_n^*\right)$ is a Nash equilibrium if,

$$s_1^* \in B_i\left(s_{-i}^*\right), \qquad \forall i = 1, \ldots, n. \qquad \textbf{(3)}$$

Given a strategic form game $\Gamma = \langle N, (S_i), (u_i) \rangle$, a strongly (weakly) dominant strategy equilibrium $\left(s_1^*, \ldots, s_n^*\right)$ is also a Nash equilibrium. The intuitive explanation for this is as follows. In a dominant strategy equilibrium, the equilibrium strategy of each player is a best response irrespective of the strategies of the rest of the players. In a pure strategy Nash equilibrium, the equilibrium strategy of each player is a best response against the Nash equilibrium strategies of the rest of the players. Thus, the Nash equilibrium is a much weaker version of a dominant strategy equilibrium. It is also fairly obvious to note that a Nash equilibrium need not be a dominant strategy equilibrium.

Furthermore, Nash's Existence Theorem states that for every game with finitely strategies there will be at least one Nash equilibrium. Thus, the concept of Nash equilibrium is not problematic. The theorem establishes that the notion of a Nash equilibrium is coherent in a deep way (JEHLE, RENY, 2001). For any finite game, it is guaranteed that there will be a Nash equilibrium solution to the strategy problem.

EXAMPLES OF NASH EQUILIBRIUM

Battle of the Sexes (BOS):

The BOS games can be exemplified with the following payoff matrix:

Table 3 - Battle of the sexes payoff

1	2	
	A	B
A	2,1	0,0
B	0,0	1,2

There are two Nash equilibria here, namely (A,A) and (B,B). The profile (A,A) is a Nash equilibrium because

$$\begin{aligned} u_1(M,M) > u_1(F,M) \\ u_2(M,M) > u_2(M,F) \end{aligned} \quad \textbf{(4)}$$

The profile (F,F) is a Nash equilibrium because

$$\begin{aligned} u_1(F,F) > u_1(M,F) \\ u_2(F,F) > u_2(F,M) \end{aligned} \quad \textbf{(5)}$$

The best response sets are given by:

$$B_1(M) = \{M\}\,; B_1(F) = \{F\}\,;\; B_2(M) = \{M\}\,;\; B_2(F) = \{F\} \quad \textbf{(6)}$$

Since $M \in B_1(M)$ *and* $M \in B_2(M)$, (M,M) is a Nash equilibrium. Similarly, since $F \in B_1(F)$ *and* $F \in B_2(F)$, (F,F) is a Nash equilibrium. The profile (M,F) is not a Nash equilibrium since

$$M \ni B_1(F)\,; F \ni B_2(M)\,; \quad \textbf{(7)}$$

Prisoner's Dilemma:

We consider the prisoner's dilemma problem, which has the following payoff matrix:

Table 4 - Prisoner's dilemma payoff

1	2	
	NC	C
NC	-2, -2	-10, -1
C	-1, -10	-5, -5

Note that (C, C) is the unique Nash equilibrium here. To see why we have to look at the best response sets:

$$B_1(C) = \{C\}\,; B_1(NC) = \{C\}\,;\; B_2(C) = \{C\}\,;\; B_2(NC) = \{C\} \qquad \textbf{(8)}$$

Since $s_1^* \in B_1\left(s_2^*\right)$ and $s_2^* \in B_2\left(s_1^*\right)$ for a Nash equilibrium, the only possible Nash equilibrium here is (C, C). This is a strongly dominant strategy equilibrium.

3.4.2. RAIFA-KALAIS-SMORODINSKY EQUILIBRIUM

Although the results of the Nash equilibrium have been extensively recognized, some authors criticized the theory proposed by John Nash. Raiffa and Luce published the book "Game and Decisions: Introduction and Critical Survey," criticizing the Nash equilibrium. The critic was centered on Nash's Independence of Irrelevant Alternatives axiom (IIA). The problem with the use of this axiom is the distortion that occurs when agents attribute different utilities to the ones that are being negotiated (ANBARCI; BOYD III, 2008, cited Barbosa, 2017).

The RKS approach was used by Monteiro (2018) and we will compare our results when obtaining the Nash equilibrium with the one published by Monteiro (2018).

3.5. MATLAB SOFTWARE

MATLAB (Matrix Laboratory) is a computer program used to perform engineering and scientific calculations. It was initially designed to perform matrix mathematics. However, it has grown over the years into a flexible computing system capable of solving, for the most part, any technical problem (CHAPMAN, 2012). MATLAB includes computation, visualization, and a programming environment. Furthermore, it provides a very extensive library of predefined functions, which makes it significantly easier to solve technical problems in MATLAB.

The literature analyzed shows us that there are several advantages in using the MATLAB software (CHAPMAN, 2012):

- Ease of Use: Programs can be easily written and modified with the built-in integrated development environment, and debugged with the MATLAB debugger;
- Platform Independence: MATLAB is supported on many different computer systems, providing a considerable measure of platform independence;
- Predefined Functions: Includes an extensive library of predefined functions that offer tested and prepackaged solutions to many basic technical tasks;
- Device-Independent Plotting: It has many integral plotting and imaging commands. The plots and images can be displayed on any graphical output device supported by a computer;
- Graphical User Interface: It allows the programmer to design sophisticated data analysis programs that can be operated by relatively inexperienced users;
- MATLAB Compiler: It converts MATLAB programs into ones that can be run on any computer without requiring a MATLAB license. It enables the users to distribute their programs to anyone who does not have MATLAB installed in their devices.

Due to the benefits mentioned above and its dissemination in engineering disciplines, MATLAB was chosen as the programming environment for implementing the problem analyzed in this project.

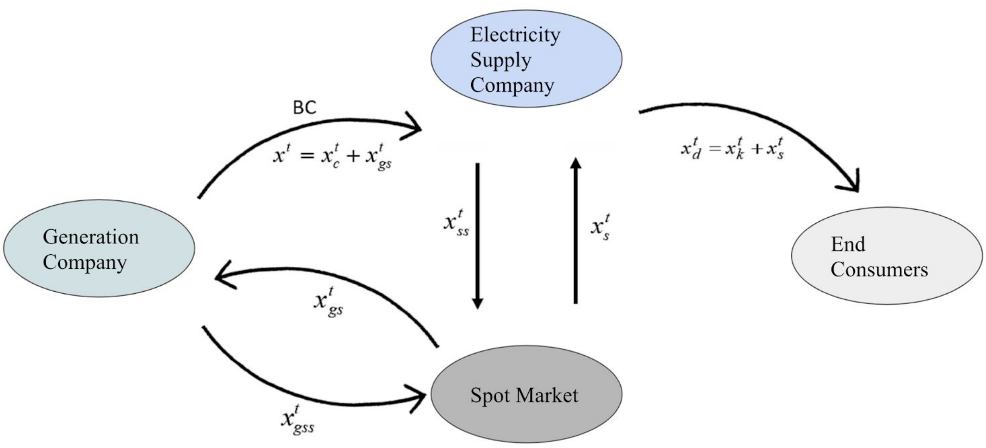
4. THE DEVELOPMENT OF THE MODEL

Bilateral contracts (BC) involves the determination of electricity amount to be delivered to the contract partner at each time interval t during the contract period (PALAMARCHUK, 2010). The involved parties are a generation company (GC) and an electricity supply company (ESC). The first represents the electricity producer, e.g., the supplier, while the second represents the electricity buyer that sells electricity to end consumers.

The contract parties usually have different interests. The supplier prefers to increase the sales in the spot market and decrease deliveries under the BC on the event of higher prices in the spot market. While under the same circumstances, the buyer wants to reduce purchase from the spot market and increase BC deliveries.

The development of this work is based on the problem presented by Palamarchuk (2010). The scenario, the model, and the numerical example are introduced in this section. Later, the model is implemented in the MATLAB software, and the numerical example is used to illustrate the methodology. The market dynamics can be illustrated as follows:

Figure 1 - Dynamic of the energy market



4.1 INDEPENDENT CONTRACT SCHEDULING

There are two types of BCs are considered as follows (PALAMARCHUK, 2010):

Table 5 - Types of bilateral contracts

Type I	The buyer (ESC) determines electricity amount to be delivered as stated in the contract at each time interval t. In this scenario, the supplier (GC) must guarantee electricity delivery according to the buyer's decision.
Type II	The supplier (GC) establishes the electricity amount to be sold under the BC at each time interval. Here, the buyer (ESC) accepts the delivered electricity in accordance with the supplier requirement.

Let ESC be going to sign a BC of type I. It determines contract deliveries over the contract period. Suppose that the contract period is divided into N equally long intervals t. The forecasts of electricity prices in the spot market at each time interval t are random variables. When scheduling the BCs the ESC maximises its expected profit S_1:

$$\max_{x_k^t, x_s^t, x_{ss}^t} S_1 = E\left\{\sum_{t=1}^{N}[p_d^t(x_k^t + x_s^t) - p_s^{\sim t} x_s^{\sim t} + p_s^{\sim t} x_{ss}^t] - J\right\} \quad \textbf{(9)}$$

Where E is a mathematical expectation symbol. Since the price J for the considered contract is constant, profit maximisation is equivalent to maximisation of the expected sales revenue R_1 in the spot and retail markets the equation can be rewritten as:

$$\max_{x_k^t, x_s^t, x_{ss}^t} R_1 = E\left\{\sum_{t=1}^{N}[p_d^t(x_k^t + x_s^t) - p_s^{\sim t} x_s^{\sim t} + p_s^{\sim t} x_{ss}^t]\right\} \quad \textbf{(10)}$$

Subject to the following constraints

a) the total contract volume:

$$\sum_{t=1}^{N} (x_k^t + x_{ss}^t) = V \qquad \textbf{(11)}$$

b) the sales to end consumers:

$$(x_k^t + x_s^t) = x_d^t, \qquad t = 1, ..., N \qquad \textbf{(12)}$$

c) the delivery amount under the contract at certain intervals:

$$x_{min}^t \leq (x_k^t + x_{ss}^t) \leq x_{max}^t, \qquad t = 1, ..., N \qquad \textbf{(13)}$$

d) the non-negativity of variables:

$$x_k^t \geq 0, x_s^t \geq 0, x_{ss}^t \geq 0, \qquad t = 1, ..., N \qquad \textbf{(14)}$$

Solving the problem, ESC determines the schedule of electricity deliveries $x^t,\ t = 1, ..., N$ for the contract period and brings it to notice of the GC. In this case, the GC is the second party of the BC. GC agrees to supply electricity according to the schedule, established by ESC.

Suppose that for each time t GC knows its production cost $C^t(x_g^t)$ as a function of electricity generation x_g^t. The function is based on the optimal unit commitment, in a way of obtaining minimum costs.

The main objective of the GC is also to maximise its profit S_2 by simultaneous participation in the spot market and the BCs market:

$$\max_{x_c^t, x_{gs}^t, x_{gss}^t} S_2 = E\left[\sum_{t=1}^{N} (p_s^{\sim t} x_{gss}^t - p_s^{\sim t} x_{gs}^t - C^t(x_c^t + x_{gss}^t)) + J\right] \qquad \textbf{(15)}$$

Since J is constant, the expression can be replaced by

$$\max_{x_c^t, x_{gs}^t, x_{gss}^t} R_2 = E\left[\sum_{t=1}^{N} (p_s^{\sim t} x_{gss}^t - p_s^{\sim t} x_{gs}^t - C^t(x_c^t + x_{gss}^t))\right] \qquad \textbf{(16)}$$

Subject to the following constraints

a) the electricity generation at each interval:

$$x^t_{gmin} \leq \left(x^t_c + x^t_{gss}\right) \leq x^t_{gmax}, \qquad t = 1, \ldots, N \tag{17}$$

b) the delivery amount under the contract at individual intervals:

$$\left(x^t_c + x^t_{gs}\right) = x^t, \qquad t = 1, \ldots, N \tag{18}$$

c) the non-negativity of variables:

$$x^t_c \geq 0,\ x^t_{gs} \geq 0, x^t_{gss} \geq 0, \qquad t = 1, \ldots, N \tag{19}$$

4.2 COMPROMISE APPROACH TO CONTRACT SCHEDULING

The two types of contracts presented in the previous section illustrate possible ways to deal with the negotiation. In both types of contract, one party has the privilege to schedule delivery while the other party plays the role of a subordinate partner. Consequently, the party who has the privilege of decision has also the greater opportunity of a higher profit. In this context, the parties seem to be in different conditions (PALAMARCHUK, 2010). However, there is a different approach in which both BC parties are seen as equal partners.

The compromise approach to contract scheduling presents another way to balance the interest of the BC parties. This approach is structured in a way that both parties agree to make an equal and minimum concession of the maximum profit, which could be obtained from their independent programming (BARBOSA, 2017). In general, this approach seeks an agreement whose contract value and electricity delivery make it profitable for both parties.

The equation that determines the maximum profit for both parties can be represented by (PALAMARCHUK, 2010):

$$S_1^* = S_2^* = R_1^* - |R_2^*|$$
$$= E\left\{\sum_{t=1}^{N}[p_d^t(x_k^{t*} + x_s^{t*}) - p_s^{\sim t}x_s^{t*} + p_s^{\sim t}x_{ss}^{t*}]\right\} - \left|E\left\{\sum_{t=1}^{N}[p_s^{\sim t}x_{gss}^{t*} - p_s^{\sim t}x_{gs}^{t*} - C^t(x_c^{t*}x_{gss}^{t*})]\right\}\right. \quad \textbf{(20)}$$

Denote by $x_s^{t0}, x_{ss}^{t0}, x_k^{t0}, x_{gs}^{t0}, x_{gss}^{t0}, x_c^{t0},\ t = 1, ..., N$ the amounts of deliveries corresponding to the compromise variant of the BC.

The revenues of ESC with compromise deliveries are:

$$R_1^0 = E\left\{\sum_{t=1}^{N}[p_d^t(x_k^{t0} + x_s^{t0}) - p_s^{\sim t}x_s^{t0} + p_s^{\sim t}x_{ss}^{t0}]\right\} \quad \textbf{(21)}$$

and the revenues of GC with compromise deliveries are:

$$R_2^0 = E\left\{\sum_{t=1}^{N}\left[p_s^{\sim t}x_{gss}^{t0} - p_s^{\sim t}x_{gs}^{t0} - C^t\left(x_c^{t0}x_{gss}^{t0}\right)\right]\right\} \quad \textbf{(22)}$$

Relative concessions of the ESC are

$$\Delta S_1^0 = \frac{1}{S_1^*}[S_1^* - (R_1^0 - J^0)] \quad \textbf{(23)}$$

Relative concessions of the GC are

$$\Delta S_2^0 = \frac{1}{S_2^*}[S_2^* - (J^0 - |R_2^0|)] \quad \textbf{(24)}$$

The Nash point from the Pareto-optimal set provides players with the maximum possible profits, at their relative concessions being equal. Denote S_1^0 and S_2^0 the profits of ESC and

GC at the Nash point. The necessary condition for obtaining the Nash point is max ($S_1^0 \, x \, S_2^0$) on the set of possible profits.

$$J^0 = \frac{1}{2}(R_1^* + |R_2^*|) \qquad \textbf{(25)}$$

Therefore, the Nash equilibrium point is the midpoint between the revenues of ESC and GC. Denote $S_1^0 = S_1^*/2$ and $S_2^0 = S_2^*/2$, the relative concessions can be rewritten as:

$$\Delta S_1^0 = \frac{1}{S_1^*}\left[S_1^* - R_1^0 + \frac{1}{2}(R_1^* + |R_2^*|)\right] \qquad \textbf{(26)}$$

$$\Delta S_2^0 = \frac{1}{S_2^*}\left[S_2^* + |R_2^0| - \frac{1}{2}(R_1^* + |R_2^*|)\right] \qquad \textbf{(27)}$$

An important aim for the compromise contract scheduling is to obtain electricity deliveries by time intervals t. The following problem can be solved for this purpose, where k represents the concession:

$$\min_{k, x_k^{t0}, x_s^{t0}, x_{ss}^{t0}, x_c^{t0}, x_{gs}^{t0}, x_{gss}^{t0},} \quad k \qquad \textbf{(28)}$$

subject to

a) the equality of relative profit decreases (relative concessions)

$$\frac{\left(S_1^* - E\{\sum_{t=1}^{N}[p_d^t(x_k^t + x_s^t) - p_s^{\sim t}x_s^t + p_s^{\sim t}x_{ss}^t]\} + \frac{1}{2}(R_1^* + |R_2^*|)\right)}{S_1^*} = k \qquad \textbf{(29)}$$

and

$$\frac{\left(S_2^* + \left|E\{\sum_{t=1}^{N}\left[p_s^{\sim t}x_{gss}^t - p_s^{\sim t}x_{gs}^t - C^t\left(x_c^t x_{gss}^t\right)\right]\}\right| - \frac{1}{2}(R_1^* + |R_2^*|)\right)}{S_2^*} = k \qquad \textbf{(30)}$$

b) the consideration of points from the set of mutual profits

$$E\left\{\sum_{t=1}^{N}[p_d^t(x_k^t + x_s^t) - p_s^{\sim t}x_s^t + p_s^{\sim t}x_{ss}^t]\right\} \geq \left|E\left\{\sum_{t=1}^{N}[p_s^{\sim t}x_{gss}^t - p_s^{\sim t}x_{gs}^t - C^t(x_c^t x_{gss}^t)]\right\}\right| \quad \textbf{(31)}$$

c) the total contract volume

$$\sum_{t=1}^{N}(x_k^t + x_{ss}^t) = V \quad \textbf{(32)}$$

d) the sales to end consumers

$$x_k^t + x_s^t = x_d^t\ ,\ t = 1,\ldots,N \quad \textbf{(33)}$$

e) the electricity generation at each interval

$$x_{g\,min}^t \leq x_c^t + x_{gss}^t \leq x_{g\,max}^t, \quad t = 1,\ldots,N \quad \textbf{(34)}$$

f) the non-negativity of variables

$$k \geq 0, x_k^t \geq 0, x_s^t \geq 0, x_{ss}^t \geq 0, x_c^t \geq 0\ x_{gs}^t \geq 0, x_{gss}^t \geq 0,\ t = 1, \ldots, N \quad \textbf{(35)}$$

4.3. NUMERICAL EXAMPLE

In order to obtain the set of all optimal Nash equilibrium solutions the same input data used by Palamarchuk (2010) were used. Table 6 and Table 7 present randomly assigned values of the energy price on the spot market, as well as their probabilities For the analyzed problem, Palamarchuk (2010) considered 3 time intervals. The total volume considered was V=145.

Table 6 - Initial data for BC scheduling made by ESC

Time intervals	t_1			t_2			t_3		
Spot price forecasts made by ESC									
spot price scenarios, p_{sj}^t, \$/MWh	10.0	10.4	10.8	11.0	11.2	11.8	11.0	11.4	11.8
expected probabilities, ξ_j^t	0.1	0.8	0.1	0.3	0.5	0.2	0.2	0.4	0.4
electricity price for end consumers, p_d^t, \$/MWh	16			16			16		
Constraints for optimisation problem									
electricity consumption by end consumers, MWh	x_d^1			x_d^2			x_d^3		
	9.8			11.4			14.5		
limits on electricity deliveries under BC, MWh	x_{min}^1		x_{max}^1	x_{min}^2		x_{max}^2	x_{min}^3		x_{max}^3
	8		60	5		68	6		62

Table 7 - Initial data for BC scheduling made by GC

Time intervals	t_1			t_2			t_3		
Spot price forecasts made by GC									
spot price scenarios, p_{sj}^t, \$/MWh	10.8	11.2	11.6	11.0	11.6	12.0	11.0	11.8	12.4
expected probabilities, ξ_j^t	0.2	0.6	0.2	0.25	0.5	0.25	0.1	0.6	0.3
production cost functions, $C^t\ (x_g^t)$, \$	$8.4 + 1.4x_g^1 + 0.4(x_g^1)^2$			$10.4 + 1.52x_g^2 + 0.44(x_g^2)^2$			$11.2 + 1.4x_g^3 + 0.32(x_g^3)^2$		
Constraints for optimisation problem									
electricity generation limits, MWh	$x_{g\,min}^1$	$x_{g\,max}^1$		$x_{g\,min}^2$	$x_{g\,max}^2$		$x_{g\,min}^3$	$x_{g\,max}^3$	
	14	50		15	60		16	65	
electricity deliveries under BC, MWh	15			68			62		

4.4 THE MODEL

Previous studies suggest the existence of more than one Nash equilibrium for the energy bilateral contracts problem (TOLEDO, 2017; CUNHA, 2017). In those studies, simulations were made with different contract values and the same input data. The results allowed the authors to conclude that there is more than one optimal solution for the current problem.

However, the previous studies did not involve an in-depth search to determine the set of optimal solutions. After an extensive literature study, the work of Pozo and Contreras (2011) was selected because it proposes a new methodology to find all Nash equilibria.

4.4.1 METHOD TO FIND MULTIPLE NASH EQUILIBRIA BY POZO AND CONTRERAS (2011)

Normally, there is more than one pure Nash equilibrium (POZO; CONTRERAS, 2011). The proposed model consists of creating "holes" in the feasible region in order to find these equilibria. The holes are centered around each Nash equilibrium found. For each newly equilibrium, a new linear constraint (hole) is added in the feasible region, as shown in the equation XX

$$\sum_{k,t,i,b} (x^{e}_{ktib} - x^{*}_{ktib})^2 + (y^{e}_{ktib} - y^{*}_{ktib})^2 \geq r^2 \qquad \textbf{(35)}$$

The radius r must be small enough so as not to lose any solutions inside the hypersphere hole, and the solution must not belong to the boundary of the hypersphere hole (POZO; CONTRERAS, 2011)

This work studies an optimization problem presented by Palamarchuk (2010) with six variables ($x_k, x_s, x_{ss}, x_c x_{gss}, x_{gs}$) for three different time periods $t = (1, 2, 3)$. Hence, the equivalent equation to be applied to the problem is

$$\sum_{1}^{3} (x_k^{e\,t} - x_k^{*\,t})^2 + (x_s^{e\,t} - x_s^{*\,t})^2 + (x_{ss}^{e\,t} - x_{ss}^{*\,t})^2 + (x_c^{e\,t} - x_c^{*\,t})^2 + (x_{gs}^{e\,t} - x_{gs}^{*\,t})^2 + (x_{gss}^{e\,t} - x_{gss}^{*\,t})^2 \geq r^2 \qquad \textbf{(36)}$$

The mathematical problem was implemented in the MATLAB software, and the results are shown in the next chapter. The program used in this work is presented in the Appendix.

5. RESULTS

As stated before, the same numerical example proposed by Palamarchuk (2010) was replicated in the model. That makes it possible to validate the model according to its results. The input data for the algorithm are presented in tables 6 and 7. The code of the MATLAB algorithm is described in Appendix A. The standard test was conducted with total volume V=145.

After running the program (Appendix A) to find all Nash equilibria with the new constraint proposed by Pozo and Contreras (2011), the solutions were obtained as shown in table 8.

The first solution is analogous with the one given by Palamarchuk (2010) for the Nash bargain solution. Eight sets of solutions were found according to the conditions of V=145. From the ninth iteration onwards, the value of the objective function was worse than the previous one. That implies that the feasible region went through exhaustion of optimal points, due to the appearance of the holes. The relative concession was **56,08%** as presented in the solution in Palamarchuk (2010).

We can confirm that the solutions presented in table 8 meet the constraints listed above. As for example, in Solution 1 where $(2,25 + 7,55 + 9,38 + 30,44 + 60,45 + 34,93) = 145$ or $(x_k^1 + x_k^2 + x_k^3 + x_{ss}^1 + x_{ss}^2 + x_{ss}^3) = 145$, which satisfy equation XX. Following the same argument, the solution numbers can be confirmed for the other constraints.

Table 8 - Nash optimal solutions for contract value V=145

Time intervals	Values of variables for ESC X_k	X_s	X_{ss}	Values of variables for GC X_c	X_{gss}	X_{gs}
Solution 1						
t_1	2,25	7,55	30,44	7,64	6,36	25,05
t_2	7,55	3,85	60,45	11,01	3,99	56,99
t_3	9,38	5,12	34,93	5,75	10,66	38,56
Solution 2						
t_1	4,58	5,22	28,11	8,73	5,27	23,96
t_2	8,58	2,82	59,42	12,22	2,78	55,78
t_3	10,90	3,60	33,40	5,41	11,00	38,90
Solution 3						
t_1	6,40	3,40	26,29	8,88	5,12	23,81
t_2	9,46	1,94	58,54	13,08	1,92	54,92
t_3	12,04	2,46	32,27	5,31	11,09	38,99
Solution 4						
t_1	8,71	1,09	23,99	10,75	3,25	21,95
t_2	4,83	6,57	63,17	11,28	3,72	56,72
t_3	6,03	8,47	38,27	1,14	15,27	43,17
Solution 5						
t_1	3,97	5,83	28,73	4,51	9,49	28,19
t_2	4,00	7,40	64,00	3,57	11,43	64,43
t_3	5,86	8,64	38,45	11,97	4,44	32,34
Solution 6						
t_1	9,14	0,66	23,56	12,99	1,01	19,72
t_2	7,03	4,37	60,95	13,52	1,48	54,46
t_3	10,05	4,45	34,26	3,70	12,74	40,61
Solution 7						
t_1	9,06	0,74	23,64	12,24	1,76	20,45
t_2	7,97	3,43	60,03	13,63	1,37	54,37
t_3	12,33	2,17	31,98	4,29	12,12	40,02
Solution 8						
t_1	7,66	2,14	25,04	8,99	5,01	23,71
t_2	10,16	1,24	57,84	13,77	1,23	54,23
t_3	12,91	1,59	31,40	5,31	11,10	39,00

To compare the Nash equilibrium with the RKS equilibrium in this scenario, the same model was implemented by Monteiro (2018). The set of RKS solutions for V=145 is presented in Table 9.

Table 9 - RKS optimal solutions for contract value V=145

	Values of variables for ESC			Values of variables for GC		
Time intervals	X_k	X_s	X_{ss}	X_c	X_{gss}	X_{gs}
Solution 1						
t_1	4,07	5,73	10,93	4,66	9,34	10,34
t_2	4,35	7,05	63,65	2,69	12,31	65,31
t_3	5,76	8,74	56,24	2,50	13,90	59,50
Solution 2						
t_1	0,99	8,81	14,01	2,12	11,88	12,88
t_2	6,94	4,46	61,06	5,27	9,73	62,73
t_3	2,13	12,37	59,87	3,77	12,63	58,23
Solution 3						
t_1	0,65	9,15	14,35	1,53	12,47	13,47
t_2	5,54	5,86	62,46	11,02	3,98	56,98
t_3	7,90	6,60	54,10	11,31	5,10	50,69
Solution 4						
t_1	0,81	8,99	14,19	6,89	7,11	8,11
t_2	7,71	3,69	60,29	9,99	5,01	58,01
t_3	8,38	6,12	53,62	3,42	12,99	58,58
Solution 5						
t_1	5,36	4,44	9,64	4,76	9,24	10,24
t_2	7,29	4,11	60,71	6,58	8,42	61,42
t_3	5,00	9,50	57,00	14,69	1,72	47,31
Solution 6						
t_1	0,25	9,55	10,93	14,85	12,24	13,32
t_2	1,41	9,99	63,65	66,55	0,71	53,66
t_3	11,58	2,92	56,24	50,35	13,01	58,51
Solution 7						
t_1	5,21	4,59	9,79	1,23	12,77	13,77
t_2	9,01	2,39	58,99	11,16	3,84	56,84
t_3	11,51	2,99	50,49	1,77	14,64	60,23
Solution 8						
t_1	9,40	0,40	33,53	0,17	34,31	42,76
t_2	0,06	11,34	55,05	0,03	14,97	55,08
t_3	14,49	0,01	32,47	16,02	0,05	30,93

In all the sets of solutions, the best alternative for both parties was the one with the Raiffa-Kalais-Smorodinsky equilibrium, compared with the Nash equilibrium one. The relative concession was **55,01%**, e.g., the best alternative for the proposed problem.

The model for the set Nash solutions was run for different inputs of different contract values. The results are presented in Appendix B.

It is observed that there is a variability on the number of solutions depending on contract volumes. Monteiro (2018) raised an interesting question in his suggestion of future studies: "Which (equilibrium), after all, would have the largest number of solutions and how different would these solutions be?" Table 10 summarizes the number of total solutions for the contract values V=135, V=140, and V=145.

Table 10 - Comparison between the amount of solutions

Contract Volume, MWh	Optimal Solutions - RKS	Optimal Solutions - NASH
135	4	4
140	6	6
145	8	8

After running the model for both equilibria, it was interesting to note that the number of optimal solutions for both the Nash equilibrium and the RKS equilibrium is the same. This implies that, even with different hole points, the method of finding the set of all optimal solutions operates in a similar way.

6.CONCLUSION

We have discussed the importance of contract scheduling within the energy market. The problem proposed by Palamarchuk, restricted to only one optimal solution was expanded by Cunha (2017) and Toledo (2017) who were able to demonstrate the existence of more than one optimal solution, under the same data and constraints.

As proposed in the introduction, the specific objectives were satisfied. The mathematical problem was developed, and we found the set of all Nash optimal solutions to the problem. Running the model for different types of contracts (V=135, V=140, and V=145), we could compare the results to the ones founded by Barbosa (2018) with the RKS equilibrium approach. Thereby, we concluded that the Nash approach provides less optimal solutions compared to the RKS.

Another information we wanted to analyze was the number of solutions for the Nash equilibrium. In this way, it was possible to analyze the flexibility that each equilibrium approach gives to the decision makers. If a higher number of possible solutions were discovered for the Nash approach, this would mean that the agents would have greater flexibility to contract negotiation. However, that was not the case. The same number of optimal solutions was found in both equilibria. Consequently, the Nash equilibrium solution is dominated by the RKS equilibrium solution in the analyzed case study both on profit and flexibility.

There was no vast difference in the concession percentage between the two studied equilibria. However, in the case of a billion dollar market, a slight change in the negotiation can represent a significant increase in the revenue of the involved actors. Our research contributes to the decision makers of the energy market system to make good decisions regarding contract planning.

Further research can improve the current project, and help the decision makers to have better choices of contract scheduling. A limitation of the current study is that the simulations consider only two actors: the generating company (GC) and the electricity supply company (ESC). Further investigation can be made taking into account multiple actors such as more GCs and ESC, or even other actors of the electricity market: Small and large consumers.

Another issue to be analyzed is to add risk to the spot price probabilities. The current study uses the numerical example of Palamarchuk (2010), which considers a given probability for the spot prices. As explained before, the spot prices are very volatile and thus represent the major uncertainty for the involved parties. This possibility can be added to the current problem so that the risk due to the uncertainty can be minimized.

Finally, an additional study could analyze the tendency of change in the number of the set of solutions by varying the contract value. As is was seen in this work, by varying the contract value a certain number of solutions is obtained. However, this number was the same is both types of equilibria.

REFERENCES

ABHYANKAR, A. R., & KHAPARDE, S. A. *Introduction to deregulation in power industry*. Report by Indian Institute of Technology*, Mumbai*, 2013.

ANDER-EGG, E. *Introduccíon a las técnicas de investigacíon social: para trabajadores sociales.* 7 ed. Buenos Aires: Humanistas, 1978.

BARBOSA, M. L. *A Solução de Barganha de Kalais-Smorodinsky aplicada à negociação de contratos bilaterais no mercado livre de eletricidade.* Universidade de Brasília, 2017.

BASAR, T. *Non-Cooperative Game Theory* - Lecture Notes. 2010. Available o <http://www.hamilton.ie/ollie/Downloads/Game.pdf>.

BENTH, F. E., & SCHMECK, M. D. *Pricing futures and options in electricity markets.* In *The Interrelationship Between Financial and Energy Markets*. Springer, Berlin, Heidelberg, 2014.

BURNS, J. M. *A treatise on markets: spot, futures, and options*. AEI Studies, 1979.

CHAO, H. P., & HUNTINGTON, H. G. (Eds.). *Designing competitive electricity markets*. Kluwer Academic Publishers, 1998.

CHAPMAN, S. J. *MATLAB programming with applications for engineers*. Cengage Learning, 2012.

GIL, Antonio Carlos. *Como elaborar projetos de pesquisa.* 4. ed. São Paulo: Atlas, 2008.

HOUCQUE, D. *Introduction to Matlab for engineering students. Northwestern University*, 1-64, 2005

INTERNATIONAL ENERGY AGENCY. *Lessons from Liberalised Electricity Markets.* Paris, 2005. Available on <https://www.iea.org/publications/freepublications/publication/WorldEnergyOutlook2016ExecExecutiveSummaryEng.pdf>

JAMSHIDI, M., KEBRIAEI, H., & SHEIKH-EL-ESLAMI, M. K. *An interval-based stochastic dominance approach for decision making in forward contracts of electricity market.* Energy, 2018.

JEHLE, G. A., & Reny, P. J. *Advanced Microeconomic Theory*. 2001.

KAUFMANN, A. *Methods and Models of Operations Research.* Prentice-Hall, 1963.

KIRSCHEN, D. S.; STRBAC, G. *Fundamentals of Power System Economics*. Hoboken, GB: Wiley. ISBN 9780470020586. 2004. Available on: <http://site.ebrary.com/lib/gwu/docDetail.action?docID=10113950>.

LEEDY, P. D. *Practical research: planning and design*. New Jersey: Prentice-Hall, 1993.

MONTEIRO, D. S. *Análise de algoritmo para soluções múltiplas de barganha de Raiffa-Kalai-Smorodinsky à negociação de contratos de energia.* Universidade de Brasília, 2018.

NARAHARI, Y. *Game Theory* - Lecture Notes, Indian Institute of Science. 2012. Available on <http://lcm.csa.iisc.ernet.in/gametheory/ln/web-ncp5-purenash.pdf>.

OECD Factbook 2015-2016: *Economic, Environmental and Social Statistics World electricity generation by source of energy: Terawatt hours (TWh)*. Available on <DOI:http://dx.doi.org/10.1787/factbook-2015-graph86-en>

PINDYCK, R. S., & RUBINFELD, D. L.. *Microeconomics*. 2013

PINTO, T., Vale, Z., Praça, I., Pires, E., & Lopes, F. *Decision support for energy contracts negotiation with game theory and adaptive learning*. Energies, 2015.

RESENDE, J. G. L (2019). *Microeconomia* - Lecture Notes, Universidade de Brasília, 2019. Available on <sites.google.com/site/jglresende/teaching/microeconomia-2---graduacao>.

SILVA, E. L. *Metodologia da pesquisa e elaboração de dissertação/Edna Lúcia da Silva, Estera Muszkat Menezes.* – 4. ed. rev. atual. – Florianópolis: UFSC, 2005.

TRIVIÑOS, A. N. S. *Introdução à pesquisa em Ciências Sociais*: a pesquisa qualitativa em educação. São Paulo: Atlas, 1987.

VARIAN, H. R. *Intermediate Microeconomics: A Modern Approach: Seventh Edition*. WW Norton & Company, 2006.

WAGNER, H. M. *Principles of Operations Research: With Applications to Managerial Decisions.* Prentice-Hall, 1975.

WANG, Y. *Introdução à Pesquisa Operacional.* 2017. Available on <https://www.youtube.com/watch?v=4EUAnzLkHFU>.

XIA, X., SHANG, N., FANG, J., JIANG, W., LIU, J., LIU, L., & DING, Y. *Management of Bilateral Contracts for Gencos Considering the Risk in Spot Market*. Energy Procedia, 2019.

APPENDIX A - Code for obtaining all optimal nash solutions

Compromiseobjective

```
function k = compromiseobjective(x)
%This algorithm is based on the paper "Dynamic programming approach to the
%bilateral contract scheduling" by Palamarchuk in Feb 2009%
%
% GC - Generation Company
% Electricity Supply Company

%********** F U N C T I O N ************%

% Function objective - Min k

%Global Variables Declaration

global ptd;
global mean_spotESC;
global mean_spotGC;
global expectedrevenueESC;
global expectedrevenueGC;


% Revenue and profit obtained by ESC with independent scheduling
rESC = 2046.30;
sESC = 248.50;

% Revenue and profit obtained by GC with independent scheduling
rGC = 1797.80;
sGC = 248.50;

% electricity price for end consumers, $/MWh
ptd = [16; 16; 16];

% Spot price forecasts made by ESC
% spot price scenarios, $/MWh
ptsjESC = [10.0 10.4 10.8; 11.0 11.2 11.8; 11.0 11.4 11.8];
% expected probabilities
csitjESC = [0.1 0.8 0.1; 0.3 0.5 0.2; 0.2 0.4 0.4];

mean_spotESC = sum(ptsjESC.*csitjESC,2);

% Spot price forecasts made by GC
% spot price scenarios, $/MWh
ptsjGC = [10.8 11.2 11.6; 11.0 11.6 12.0; 11.0 11.8 12.4];
csitjGC = [0.2 0.6 0.2; 0.25 0.5 0.25; 0.1 0.6 0.3];
```

```
mean_spotGC = sum(ptsjGC.*csitjGC,2);

% Contract price in the compromise approach
J = (rESC + rGC)/2;

expectedrevenueESC = ptd(1)*(x(1)+x(4))-mean_spotESC(1,1)*x(4)+mean_spotESC(1,1)*x(7)... %t1
     +ptd(2)*(x(2)+x(5))-mean_spotESC(2,1)*x(5)+mean_spotESC(2,1)*x(8).... %t2
    +ptd(3)*(x(3)+x(6))-mean_spotESC(3,1)*x(6)+mean_spotESC(3,1)*x(9);... %t3

expectedrevenueGC = -(mean_spotGC(1,1)*x(13) - mean_spotGC(1,1)*x(16) - (8.4
+1.4*(x(10)+x(13))+0.4*((x(10)+x(13))^2)) ... %t1
    + mean_spotGC(2,1)*x(14) - mean_spotGC(2,1)*x(17) - (10.4 +1.52*(x(11)+x(14))+0.44*((x(11)+x(14))^2))
... %t2
                      + mean_spotGC(3,1)*x(15) - mean_spotGC(3,1)*x(18) - (11.2 +
1.4*(x(12)+x(15))+0.32*((x(12)+x(15))^2)));

k = (((sESC - ... %S1*
    (expectedrevenueESC)...
    +J)/sESC)... %J contract value
    + (sGC + ... %S2*
    + expectedrevenueGC...
    - J)/sGC)/2
```

Nonlconpaper

```
function [c, ceq] = nonlconpaper(x)


%Global Variables Declaration

global ptd;
global mean_spotESC;
global mean_spotGC;
global expectedrevenueESC;
global expectedrevenueGC;
global i;
global sym_matrix;
global x1 x2 x3 x4 x5 x6 x7 x8 x9 x10 x11 x12 x13 x14 x15 x16 x17 x18;


% Spot price forecasts made by ESC
% spot price scenarios, $/MWh
ptsjESC = [10.0 10.4 10.8; 11.0 11.2 11.8; 11.0 11.4 11.8];
% expected probabilities
csitjESC = [0.1 0.8 0.1; 0.3 0.5 0.2; 0.2 0.4 0.4];

mean_spotESC = sum(ptsjESC.*csitjESC,2);

% Spot price forecasts made by GC
```

```
% spot price scenarios, $/MWh
ptsjGC = [10.8 11.2 11.6; 11.0 11.6 12.0; 11.0 11.8 12.4];
csitjGC = [0.2 0.6 0.2; 0.25 0.5 0.25; 0.1 0.6 0.3];
mean_spotGC = sum(ptsjGC.*csitjGC,2);

% electricity consumption by end consumers, MWh
xtd = [9.8; 11.4; 14.5];

% electricity generation limits, MWh
xtgminmax = [14 50; 15 60; 16 65];

% limits on electricity deliveries under BC, MWh
xtminmax = [8 60; 5 68; 6 62];

% electricity price for end consumers, $/MWh
ptd = [16; 16; 16];

expectedrevenueESC = ptd(1)*(x(1)+x(4))-mean_spotESC(1,1)*x(4)+mean_spotESC(1,1)*x(7)... %t1
    +ptd(2)*(x(2)+x(5))-mean_spotESC(2,1)*x(5)+mean_spotESC(2,1)*x(8).... %t2
   +ptd(3)*(x(3)+x(6))-mean_spotESC(3,1)*x(6)+mean_spotESC(3,1)*x(9);... %t3

expectedrevenueGC = -(mean_spotGC(1,1)*x(13) - mean_spotGC(1,1)*x(16) - (8.4
+1.4*(x(10)+x(13))+0.4*((x(10)+x(13))^2)) ... %t1
   + mean_spotGC(2,1)*x(14) - mean_spotGC(2,1)*x(17) - (10.4 +1.52*(x(11)+x(14))+0.44*((x(11)+x(14))^2))
... %t2
                    + mean_spotGC(3,1)*x(15) - mean_spotGC(3,1)*x(18) - (11.2 +
1.4*(x(12)+x(15))+0.32*((x(12)+x(15))^2)));

c(1)= -expectedrevenueESC + expectedrevenueGC; %constraint (31)

% Revenue and profit obtained by ESC with independent scheduling
rESC = 2046.30;
sESC = 248.50;

% Revenue and profit obtained by GC with independent scheduling
rGC = 1797.80;
sGC = 248.50;

% Contract price in the compromise approach
J = (rESC + rGC)/2;

ceq = -expectedrevenueESC/sESC - expectedrevenueGC/sGC + 2*J/sESC;


if i>=2


subs_calculation = double(subs(sym_matrix, {x1, x2, x3, x4, x5,
x6,x7,x8,x9,x10,x11,x12,x13,x14,x15,x16,x17,x18},
{x(1),x(2),x(3),x(4),x(5),x(6),x(7),x(8),x(9),x(10),x(11),x(12),x(13),x(14),x(15),x(16),x(17),x(18)}));
```

```
z=size(c(1));
s=size(subs_calculation);
d = [c(1);zeros(z(1)-s(1),1)];
c = vertcat(d, subs_calculation);

end

end
```

Compromiseapproachpaper

```
clc;
clear all;

%Global Variables Declaration

global ptd;
global mean_spotESC;
global mean_spotGC;
global expectedrevenueESC;
global expectedrevenueGC;
global i;
global testei;
global r;
global sym_matrix;
global x1 x2 x3 x4 x5 x6 x7 x8 x9 x10 x11 x12 x13 x14 x15 x16 x17 x18;

%amount of electricity delivered under BC (V)
V=170;
maxProfitESC = 283.1; % According to independent scheduling made by ESC;
maxProfitGC = 266.1; % According to independent scheduling made by GC

%electricity price for end consumers, $/MWh (ptd)
ptd = [16;16;16];

%spot price scenarios
ptsjESC = [10.0 10.4 10.8; 11.0 11.2 11.8; 11.0 11.4 11.8];

csitjESC = [0.1 0.8 0.1; 0.3 0.5 0.2; 0.2 0.4 0.4];

mean_spotESC = sum(ptsjESC.*csitjESC,2);

ptsjGC = [10.8 11.2 11.6; 11.0 11.6 12.0; 11.0 11.8 12.4];

csitjGC = [0.2 0.6 0.2; 0.25 0.5 0.25; 0.1 0.6 0.3];

mean_spotGC = sum(ptsjGC.*csitjGC,2);
% electricity consumption by end consumers, MWh
xtd = [9.8; 11.4; 14.5];
```

```
% electricity generation limits, MWh
xtgminmax = [14 50; 15 60; 16 65];

% limits on electricity deliveries under BC, MWh
xtminmax = [8 60; 5 68; 6 62];

x0 = [9.8 11.9 14.5 0 0 0 22.9 56.6 29.8 9.5 15 5.12 4.5 0 11.25 23.2 53 39.18]; %original solution


A = [0 0 0 0 0 0 0 0 0 1 0 0 1 0 0 0 0 0 0;...
     0 0 0 0 0 0 0 0 0 0 1 0 0 1 0 0 0 0 0;...
     0 0 0 0 0 0 0 0 0 0 0 1 0 0 1 0 0 0 0;... %(34) maxgeneration
     0 0 0 0 0 0 0 0 0 -1 0 0 -1 0 0 0 0 0 0;...
     0 0 0 0 0 0 0 0 0 0 -1 0 0 -1 0 0 0 0 0;...
     0 0 0 0 0 0 0 0 0 0 0 -1 0 0 -1 0 0 0 0;... %(34) mingeneration
     1 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0; ...
     0 1 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0;...
     0 0 1 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0;... %(36) maxdelivery
     -1 0 0 0 0 0 -1 0 0 0 0 0 0 0 0 0 0 0 0; ...
     0 -1 0 0 0 0 0 -1 0 0 0 0 0 0 0 0 0 0 0;...
     0 0 -1 0 0 0 0 0 -1 0 0 0 0 0 0 0 0 0 0;... %(36) mindelivery
     -eye(19)]; %(37)

 b = [xtgminmax(1,2);...
      xtgminmax(2,2);...
      xtgminmax(3,2);...
      -xtgminmax(1,1);...
      -xtgminmax(2,1);...
      -xtgminmax(3,1);...
      xtminmax(1,2);...
      xtminmax(2,2);...
      xtminmax(3,2);...
      -xtminmax(1,1);...
      -xtminmax(2,1);...
      -xtminmax(3,1);...
      zeros(19,1)];

  Aeq = [1 1 1 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0;... %(32)
         1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0;...
         0 1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0;...
         0 0 1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0;... %(33)
         1 0 0 0 0 0 1 0 0 -1 0 0 0 0 0 -1 0 0 0;...
         0 1 0 0 0 0 0 1 0 0 -1 0 0 0 0 0 -1 0 0;...
         0 0 1 0 0 0 0 0 1 0 0 -1 0 0 0 0 0 -1 0]; %(35)

  beq = [V;
         xtd(1);
         xtd(2);
         xtd(3);
         0;
```

```
        0;
        0];


    lb = [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0];
    ub = [V V V V V V V V V V V V V V V V V V 2000];

    r = 1;

    x0 = [9.8 11.4 14.5 0 0 0 22.9 56.6 29.8 9.5 15 5.12 4.5 0 11.25 23.2 53 39.18 1492.5];

    for i=1:9


      if i<=1
          fprintf('\n***Solving for Min k ***\n\tCalculating, please wait..');
    [x,fval]=fmincon(@compromiseobjective,x0,A,b,Aeq,beq,lb,ub,@nonlconpaper);

    k = fval;

    %x = fmincon(@compromiseobjective,x0,A,b,Aeq,beq,lb,ub,@nonlconpaper,[]);

    solutionESC = [x(1) x(4) x(7); x(2) x(5) x(8); x(3) x(6) x(9)]
    solutionGC = [x(10) x(16) x(13); x(11) x(17) x(14); x(12) x(18) x(15)]

 % by using a (slightly) different initial point x0 (i.e [9 11 14 0 0 0 22 56 29 9 15 5 4 0 11 23 53 39])
% to minimize our problem, we still find the same value for k = 0.5608.
% However, by doing this, we get to another optimal solution, what suggests
% that the compromise scheduling (and not only the independent scheduling)
% also presents multiple optimal solutions.


xmatrix(i,:)=x;

clearvars x

testei(1,:) = xmatrix(1,:);

      else if i>=2

 syms x1 x2 x3 x4 x5 x6 x7 x8 x9 x10 x11 x12 x13 x14 x15 x16 x17 x18

constraint_matrix = sym(-((((x1)-testei(i-1,1))^2+((x2)-testei(i-1,2))^2
+((x3)-testei(i-1,3))^2+((x4)-testei(i-1,4))^2+ ...
        ((x5)-testei(i-1,5))^2+((x6)-testei(i-1,6))^2+((x7)-testei(i-1,7))^2+((x8)-testei(i-1,8))^2+ ...
        ((x9)-testei(i-1,9))^2+((x10)-testei(i-1,10))^2+((x11)-testei(i-1,11))^2+((x12)-testei(i-1,12))^2+ ...
        ((x13)-testei(i-1,13))^2+((x14)-testei(i-1,14))^2+((x15)-testei(i-1,15))^2+((x16)-testei(i-1,16))^2+ ...
        ((x17)-testei(i-1,17))^2+((x18)-testei(i-1,18))^2)) + r^2);

sym_matrix{i-1,1} = (constraint_matrix);
```

```
fprintf('\n***Solving for Min k*** \n \tCalculating, please wait...');
[x,fval] = fmincon(@compromiseobjective,x0,A,b,Aeq,beq,lb,ub,@nonlconpaper);
solutionESC = [x(1) x(4) x(7); x(2) x(5) x(8); x(3) x(6) x(9)];
solutionGC = [x(10) x(16) x(13); x(11) x(17) x(14); x(12) x(18) x(15)];
k = fval;
xmatrix(i,:) = x;
testei(i,:) = xmatrix(i,:);

end
end
end


   expectedrevenueESC = ptd(1)*(x(1)+x(4))-mean_spotESC(1,1)*x(4)+mean_spotESC(1,1)*x(7)... %t1
  +ptd(2)*(x(2)+x(5))-mean_spotESC(2,1)*x(5)+mean_spotESC(2,1)*x(8).... %t2
  +ptd(3)*(x(3)+x(6))-mean_spotESC(3,1)*x(6)+mean_spotESC(3,1)*x(9)... %t3

expectedrevenueGC = -(mean_spotGC(1,1)*x(13) - mean_spotGC(1,1)*x(16) - (8.4
+1.4*(x(10)+x(13))+0.4*((x(10)+x(13))^2))... %t1
  + mean_spotGC(2,1)*x(14) - mean_spotGC(2,1)*x(17) - (10.4 +1.52*(x(11)+x(14))+0.44*((x(11)+x(14))^2))
... %t2
                  + mean_spotGC(3,1)*x(15) - mean_spotGC(3,1)*x(18) - (11.2 +
1.4*(x(12)+x(15))+0.32*((x(12)+x(15))^2)))

% Revenue and profit obtained by ESC with independent scheduling
rESC = 2046.30;
sESC = 248.50;

% Revenue and profit obtained by GC with independent scheduling
rGC = 1797.80;
sGC = 248.50;

J=1650.61    %CHANGED BY REINALDO - 17 AUGUST 2017

concessionESC = (sESC - expectedrevenueESC + J)/sESC
concessionGC = (sGC + expectedrevenueGC - J)/sGC
```

APPENDIX B – Nash optimal solutions

Table 11 - Nash optimal solutions for contract value V=135

	Values of variables for ESC			Values of variables for GC		
Time intervals	X_k	X_s	X_{ss}	X_c	X_{gss}	X_{gs}
			Solution 1			
t_1	3,93	5,87	24,16	7,33	6,67	20,77
t_2	4,38	7,02	63,62	8,13	6,87	59,87
t_3	9,72	4,78	29,18	4,51	11,89	34,39
			Solution 2			
t_1	4,76	5,04	23,33	7,53	6,47	20,56
t_2	7,31	4,09	60,69	11,00	4,00	57,00
t_3	10,19	4,31	28,72	4,04	12,37	34,87
			Solution 3			
t_1	1,81	7,99	26,29	7,37	6,63	20,72
t_2	10,39	1,01	57,61	13,89	1,11	54,11
t_3	11,88	2,62	27,02	3,84	12,56	35,06
			Solution 4			
t_1	5,83	3,97	22,26	7,85	6,15	20,24
t_2	9,14	2,26	58,86	12,78	2,22	55,22
t_3	11,19	3,31	27,72	3,60	12,81	35,30

Table 12 - Nash optimal solutions for contract value V=140

	Values of variables for ESC			Values of variables for GC		
Time intervals	X_k	X_s	X_{ss}	X_c	X_{gss}	X_{gs}
			Solution 1			
t_1	2,24	7,56	17,60	4,78	9,22	15,06
t_2	7,48	3,92	60,52	12,81	2,19	55,19
t_3	6,28	8,22	45,88	8,19	8,22	43,97
			Solution 2			
t_1	1,62	8,18	18,22	4,70	9,30	15,14
t_2	6,80	4,60	61,20	12,52	2,48	55,48
t_3	2,77	11,73	49,39	8,27	8,14	43,89
			Solution 3			
t_1	1,28	8,52	18,56	4,67	9,33	15,17
t_2	6,46	4,94	61,54	12,32	2,68	55,68
t_3	1,11	13,39	51,05	8,30	8,11	43,86
			Solution 4			
t_1	1,71	8,09	18,12	4,73	9,27	15,11
t_2	6,83	4,57	61,17	12,57	2,43	55,43
t_3	1,85	12,65	50,31	8,24	8,17	43,92
			Solution 5			
t_1	4,42	5,38	15,42	5,13	8,87	14,71
t_2	9,45	1,95	58,55	13,84	1,16	54,16
t_3	11,50	3,00	40,66	7,87	8,54	44,29
			Solution 6			
t_1	3,95	5,85	15,89	5,06	14,78	8,94
t_2	8,97	2,43	59,03	13,54	54,46	1,46
t_3	8,79	5,71	43,37	7,93	44,23	8,48

APPENDIX C - RKS optimal solutions

Table 13 - RKS optimal solutions for contract value V=135

	Values of variables for ESC			Values of variables for GC		
Time intervals	X_k	X_s	X_{ss}	X_c	X_{gss}	X_{gs}
			Solution 1			
t_1	3,37	6,43	4,63	2,66	11,34	5,34
t_2	9,55	1,85	58,45	11,29	3,71	56,71
t_3	12,93	1,57	46,07	15,39	1,01	43,61
			Solution 2			
t_1	5,07	4,73	2,93	5,09	8,91	2,91
t_2	1,58	9,82	66,42	4,57	10,43	63,43
t_3	8,01	6,49	50,99	1,76	14,64	57,24
			Solution 3			
t_1	7,36	2,44	0,64	2,10	11,90	5,90
t_2	3,13	8,27	64,87	2,84	12,16	65,16
t_3	3,21	11,29	55,79	9,64	6,77	49,36
			Solution 4			
t_1	2,39	7,41	5,61	5,42	8,58	2,58
t_2	6,01	5,39	61,99	4,93	10,07	63,07
t_3	9,48	5,02	49,52	14,56	1,84	44,44

Table 14 - RKS optimal solutions for contract value V=140

	Values of variables for ESC			Values of variables for GC		
Time intervals	X_k	X_s	X_{ss}	X_c	X_{gss}	X_{gs}
			Solution 1			
t_1	2,93	6,87	7,07	4,39	9,61	5,61
t_2	6,32	5,08	61,68	7,49	7,51	60,51
t_3	9,14	5,36	52,86	0,72	15,69	61,28
			Solution 2			
t_1	7,59	2,21	2,41	4,25	9,75	5,75
t_2	1,44	9,96	66,56	10,92	4,08	57,08
t_3	3,51	10,99	58,49	10,88	5,52	51,12
			Solution 3			
t_1	0,53	9,27	9,47	2,52	11,48	7,48
t_2	4,75	6,65	63,25	0,60	14,40	67,40
t_3	1,53	12,97	60,47	11,04	5,37	50,96
			Solution 4			
t_1	0,82	8,98	9,18	3,52	10,48	6,48
t_2	8,96	2,44	59,04	3,80	11,20	64,20
t_3	4,85	9,65	57,15	1,98	14,43	60,02
			Solution 5			
t_1	4,95	4,85	5,05	3,49	10,51	6,51
t_2	9,25	2,15	58,75	8,82	6,18	59,18
t_3	6,38	8,12	55,62	13,29	3,11	48,71
			Solution 6			
t_1	9,30	0,50	0,70	3,77	10,23	6,23
t_2	2,46	8,94	65,54	2,29	12,71	65,71
t_3	2,89	11,61	59,11	0,78	15,62	61,22