TRABALHO DE GRADUAÇÃO EM ENGENHARIA ELÉTRICA

Application of the Unscented Transform to Time-Difference of Arrival Estimation of Signals of Global Navigation Satellite Systems (GNSS)

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Brasília, Julho 2016

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Application of the Unscented Transform to Time-Difference of Arrival Estimation of Signals of Global Navigation Satellite Systems (GNSS)

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Dedicatória

Aos meus amados pais Socorro & Galba, à minha irmã Rebecca e a todos os meus professores e colegas de curso que também fizeram parte da minha jornada até aqui.

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RESUMO

O presente trabalho apresenta uma aplicação da Transformada da Incerteza na estimativa do tempo de atraso entre um sinal enviado pelo satélite de sistema GNSS, e o mesmo sinal no momento em que é recebido pela antena de um receptor. O processo de identificação e posterior *tracking* do sinal recebido é necessário para a leitura dos dados codificados no sinal enviado pelo satélite. A partir disso, o sistema de navegação por satélites (destes o mais conhecido o GPS) se torna a ferramenta indispensável que é hoje. Através das informações contidas no código do sinal é possível ter conhecimento de localização, tempo e velocidade de pessoas e objetos no espaço. Para que isso ocorra, ao ser recebido, o receptor gera cópias atrasadas desse sinal e as correlaciona com o sinal que chega. A função correlação do sinal é feita de maneira discreta e em pontos escolhidos arbitrariamente. Esse procedimento é padrão e necessário para a sincronização com o sinal sendo enviado pelo satélite. Este trabalho então, propõe que a discretização da função correlação do sinal seja realizada a partir da Transformada da Incerteza. Os pontos Sigmas obtidos por meio do uso da transformada mantém as propriedades estatísticas da função densidade de probabilidade do sinal recebido, tornando a discretização um processo não mais arbitrário.

Palavras Chave: Transformada da Incerteza, Discretização, Correlação, Teoria da Probabilidade, sinais GNSS.

ABSTRACT

This work details the application of the Unscented Transform in the time-delay estimation between the signal sent by the satellite and, the same signal when it is received at the transmitter's antenna. The procedure of identifying the satellite ID and then tracking the signal is an important task in the translation of the information coded in it. Through the encrypted information on the satellite signal, it is possible to estimate localization, time and velocity of people and objects in the space. That is also why satellite navigation systems (from those GPS being the most known) are essential nowadays. To this purpose, as the signal arrives, different time-delayed copies of the same signal are generated and then correlated with the incoming signal. Currently, the discrete correlation function of the signal is analyzed at especific arbitrary points. That is a standard procedure and also necessary to complete the Time-Difference of Arrival estimation. This work proposes a new way to discretize the signal correlation function by using the Unscented Transform (UT). The Sigma points obtained through the UT keep the estatistical properties of the signal's probability density function, making that the chosen time-shift delays to perform the correlation yeld not an arbitrary process any more.

Keywords: Unscented Transform, Discretization, Correlation, Probability Theory, GNSS signals.

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Abbreviations

AWGN	Additive White Gaussian Noise.
BPSK	Binary Phase-Shift Keying.
C/A	Coarse/Acquisition.
CDF	Cumulative Distribution Function.
CDMA	Code Division Multiple Access.
GNSS	Global Navigation Satellite System.
GPS	Global Positioning System.
LOS	Line of Sight.
NLOS	Non-Line of Sight.
PDF	Probability Density Function.
PRN	Pseudo-Random Noise.
RHCP	Right Hand Circular Polarization.
RV	Random Variable.
SD	Standard Deviation.
SNR	Signal-to-Noise Ratio.
SVID	Satellite-Vehicle Identification Number.
TDOA	Time-Difference of Arrival.
UT	Unscented Transform.

1 Sumário

Sistemas de navegação via satélite fazem parte de grande parte das atividades que permeiam a vida diária de muitas pessoas. Aplicações desde as mais óbvias, como na aviação e para fins de deslocamento de pessoas e veículos em lugares desconhecidos, as informações enviadas por satélites também servem para a agricultura, monitoramento e controle do meio ambiente, bem como na previsão de possíveis fenômenos da natureza.

O Sistema de Posicionamento Global (GPS) [9] é o sistema de navegação via satélite mais utilizado. Ele é operado e controlado pelos Estados Unidos da América, o qual se compromete a manter pelo menos 24 satélites disponíveis 95 % do tempo. Os satélites do sistema GPS estão distribuídos em seis órbitas e permitem que receptores desses sinais informem a seus usuários sobre posição, velocidade, tempo e direção de pessoas e objetos em qualquer lugar do espaço.

Os sinais de GPS, os quais foram usados para as simulações de sinais GNSS deste trabalho, utilizam a técnica do espalhamento espectral (*Spread Spectrum*). A partir dela, o sinal enviado ocupa uma largura de banda bem maior do que a necessária para conter a informação nele codificada. Por meio dessa técnica, o sinal apresenta potência dispersa em grande largura de banda, o que o torna difícil de ser detectado, e a redundância espectral faz com que o sinal seja resistente a interferências externas (*jammings*). Uma outra importância do espalhamento espectral em sinais de satélite é possibilitar que outros satélites, cada um usando um código pseudo-aleatório diferente, também enviem seus sinais na mesma banda de frequência sem que isso interfira na mensagem codificada. Tal característica permite que vários transmissores enviem seus sinais ao mesmo tempo e é chamada de Acesso Múltiplo por Divisão de Código, do termo em inglês *Code Division Multiple Access* (CDMA).

Os sinais de GPS são continuamente enviados através da modulação por deslocamento de fase (ou BPSK do inglês *Binary Phase-Shift Keying*) à taxa de 50 bit/s. Os dados modulados no sinal são enviados em dois códigos pseudo-aleatórios (ou PRN – *Pseudo-Random Noise*) de espectro alargado: um no modo preciso (P) e reservado para uso exclusivo militar, e um outro de aquisição inicial (C/A – *Coarse Acquisition Code*). O código C/A é liberado para uso público e trata-se de uma sequência pseudoaleatória formada por 1023 chips enviados à taxa de 1.023 Mchips/s.

Um receptor de sinal GNSS demodula o sinal recebido e detecta as diferenças entre o sinal transmitido e a réplica do código identificado, a qual é gerada pelo próprio receptor. Esse processo de gerar cópias do sinal recebido e em seguida correlacioná-las com o sinal que chega é a chave de funcionamento de um receptor de sinal de satélites. O conceito da correlação usado para medir a semelhança entre os sinais é o que permite ao receptor se sincronizar ao sinal transmitido pelo satélite em órbita. Esse mecanismo é utilizado durante a fase de aquisição do sinal no receptor e tem o objetivo de obter o TDOA, do inglês *Time-Difference of Arrival*. Tal parâmetro representa o tempo que o sinal viajou desde emissor até o receptor.

A partir da função de correlação do sinal, a decisão de se o processo de aquisição está completo ou se ainda se faz necessário gerar réplicas do sinal com outras combinações de fase e frequência Doppler é feita através da análise de apenas alguns pontos da função adquirida. Essa discretização é realizada atualmente em pontos fixos, os quais foram definidos de maneira arbitrária e se mantém a cada correlação.

Motivação deste Trabalho

Diante da importância e utilidade de sinais GNSS, o presente trabalho propõe a aplicação da Transformada da Incerteza (UT – *Unscented Transform*) na escolha dos pontos discretos da função correlação calculada durante a fase de aquisição do sinal. A UT é uma ferramenta de discretização, que através de uma transformação não-linear, fornece pontos que pertencem à função original. Estes pontos, chamados de sigmas, quando multiplicados por seus respectivos pesos, mantem os momentos estatísticos da função de origem. Dessa forma, esses pontos guardam características da função densidade de probabilidade real do sinal que chega ao receptor. O uso desses pontos sigmas durante a etapa de processamento de banda, dentro do receptor GNSS, deixaria de apresentar uma justificativa arbitrária na escolha dos seus pontos discretos da função correlação, passando a assumir uma escolha de caráter determinístico.

Apesar de ter sido proposta há aproximadamente vinte anos, a UT tem sido aplicada em vários campos da engenharia elétrica e se apresenta como uma ótima ferramenta na transformação da função densidade de probabilidade de uma variável aleatória em uma versão discreta dela mesma, ao mesmo tempo que preserva os momentos estatísticos da distribuição. Basicamente, o objetivo da UT é aproximar um mapeamento não-linear de uma variável contínua aleatória por meio de um conjunto de pontos selecionados deterministicamente. O seu uso estudado e simulado neste trabalho apresenta-se como algo novo, não havendo a autora encontrado registro do uso da UT nesse sentido.

Apresentada a motivação deste trabalho, o seu desenvolvimento está organizado de maneira a se atingir o objetivo porposto. No Capítulo 3 conceitos da teoria da probabilidade são apresentados, assim como a forma de operação de um receptor de sinais GNSS. Segue-se então o Capítulo 4 com a descrição matemática do modelo de dados utilizado neste trabalho. O Capítulo 5 discorre sobre a UT e suas equações. Os Capítulos 6 e 7 mostram a aplicação da UT na estimação do tempo de diferença entre o sinal enviado e o mesmo sinal, quando recebido pelo receptor. Os resultados adquiridos neste trabalho podem ser encontrados no Capítulo 8. Conclusões e perspectivas de trabalhos futuros são o tema do Capítulo 9.

Descrição do Modelo Proposto

O modelo de dados aplicado a este trabalho, assume que L ondas de banda estreita são recebidas por um sistema formado por uma antena. O sinal recebido de um satélite em um instante t pode ser descrito matematicamente como:

$$x(t) = s(t) + n(t) = \sum_{l=1}^{d} s_l(t) + n(t) \tag{1.1}$$

em que $s(t) \in \mathbb{C}^{L\times 1}$ refere-se às réplicas sobrepostas do sinal que chega

$$s_l(t) = \gamma_l e^{j2\pi\nu_l t} c(t - \tau_l) \tag{1.2}$$

onde, γ_l é a amplitude complexa, ν_l a frequência Doppler, e $c(t - \tau_l)$ representa a sequência $c(t)$ do código pseudo-aleatório, o qual é periodicamente repetido com atraso τ_l, chip de duração T_c e período $T = N_c T_c$, com $N_c \in \mathbb{N}$ representando o número de chips. O ruído presente no instante de tempo t é representado por $n(t)$. Foi assumido o ruído aditivo Gaussiano branco (do inglês *Additive White Gaussian Noise* ou AWGN), o qual segue a distribuição normal padrão $\mathcal{N}(0,\sigma^2)$ com média zero e variância σ^2. O parâmetro para sinais de linha divisada (LOS – *Line of Sight*) são indicados com $l = 1$ e os parâmetros para sinais de multipercurso (NLOS – *Non-Line of Sight*), com $l = 2, \ldots, L$. Uma vez que cada satélite é identificado através de uma sequência única de código PRN, não foi considerado necessário definir $s_l(t)$ em termos do satélite. A Figura 1.1 faz uma ilustração do modelo descrito.

Transformada da Incerteza

A Transformada da Incerteza foi desenvolvida por Julier e Uhlman em 1997 [4], [5]. Em [4] Julier apresenta a UT como um "filtro de distribuição aproximada", cuja teoria matemática é baseada na estimativa dos momentos estatísticos da distribuição. Uma possível maneira de se realizar isso é aproximar uma distribuição contínua $w(\hat{u})$ de uma variável aleatória $\hat{u}$ por uma distribuição discreta $w_i(\hat{u})$, existindo apenas em pontos discretos S_i. A aproximação é implementada de forma que o mapeamento das duas distribuições apresente os mesmos momentos depois do mapeamento não-linear.

Sejam os momentos de uma distribuição contínua serem dados por:

$$E_c\left\{\hat{u}^k\right\} = \int_{-\infty}^{\infty} \hat{u}^k w(\hat{u}) \tag{1.3}$$

Ao passo que a distribuição discreta é definida em alguns pontos selecionados S_i, chamados sigmas. A partir daí os momentos se tornam:

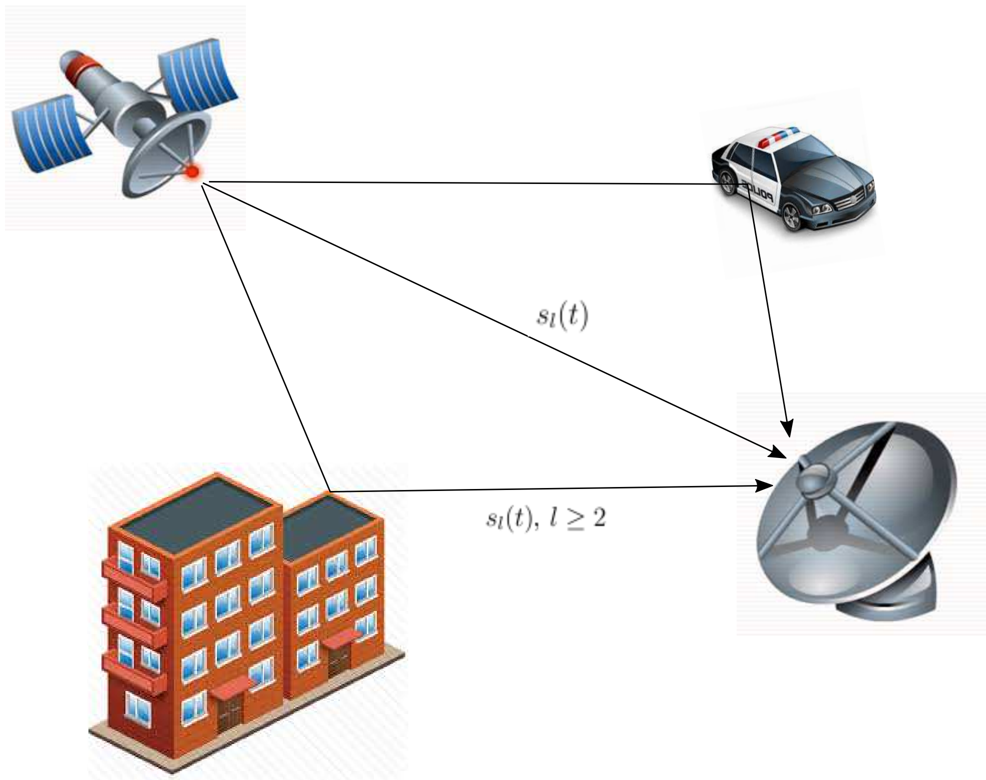

Figura 1.1: Esquemático do modelo de dados utilizado neste trabalho. Um sistema formado por uma antena recebe sinais de linha divisada (LOS) e de multipercurso (NLOS). (Ícones usados na Figura encontrados nas páginas da internet: iconfinder.com; iconexperience.com e findicons.com).

$$E_d\left\{\hat{u}^k\right\} = \int_{-\infty}^{\infty} \hat{u}^k w_i\left(\hat{u}\right) d\hat{u} = \int_{-\infty}^{\infty} \hat{u}^k \left[\sum_i w_i \delta\left(\hat{u} - S_i\right)\right] d\hat{u} = \sum_t w_i S_i^k \tag{1.4}$$

Os pesos w_i e pontos sigmas S_i são calculados a partir da Equação (1.3). Assim, o mapeamento não-linear é representado como:

$$\sum_i w_i G\left(S_i\right) = \int_{-\infty}^{\infty} G\left(\hat{u}\right) w\left(\hat{u}\right) d\hat{u} \tag{1.5}$$

A equação (1.5) apresenta a expressão da UT ao valor esperado como uma aproximação da integral da função do mapeamento não linear $G\left(\hat{u}\right)$ pesado por uma função $w\left(\hat{u}\right)$. O mapeamento não-linear refere-se ao processo que afeta a variável aleatória $\hat{u}$. O processo pode ser tanto analítico quanto numericamente computacional.

É possível transformar a série discreta em um polinômio de Taylor e truncá-lo de maneira que se encontre o valor dos pontos sigmas S_i e pesos w_i. No fim do processo de truncamento, chega-se a um grupo único de equações que representam as condições da UT:

$$w_0 = 1 - \sum_i w_i \tag{1.6}$$

$$\sum_i w_i S_i^k = E\left\{\hat{u}^k\right\} \tag{1.7}$$

A equação (1.7), deixa claro que para se aplicar a UT faz-se necessário ter conhecimento dos momentos estatísticos da distribuição da variável aleatória em questão. Uma vez que essa informação é obtida, a Transformada da Incerteza pode ser usada, mesmo desconhecendo-se o tipo de função densidade de probabilidade seguida pela variável.

Receptor de Sinais GNSS como Parte do Objetivo deste Trabalho

A palavra-chave que define a operação de um receptor de sinais GNSS é sincronização. Sincronizar com o sinal que chega, gerar cópias observáveis deste sinal, e ter acesso à mensagem de navegação que será usada no fornecimento das soluções esperadas pelo usuário, constituem as atividades básicas deste aparelho.

Uma vez que um sinal é verificado, o receptor reconhece o número que identifica o satélite através da correlação entre códigos PRN conhecidos e o sinal que chega. Ao ser exatamente identificado, tem-se início a um *loop* que estima o atraso na fase do código do sinal, bem como correções ao efeito da frequência Doppler. Essa estimativa é realizada através de um banco de correlatores, os quais geram réplicas do sinal identificado, porém com diferentes arranjos de fase do código e de frequência Doppler. Esses bancos de correlatores calculam continuamente correlações entre o sinal original e sua cópia deslocada no tempo até que se atinja o máximo da função correlação, o que indicará sincronização entre os dois sinais. Todo esse processo refere-se à fase de Aquisição do sinal e ocorre na parte de processamento de dados no receptor.

Os receptores de sinais GNSS lidam com sinais digitalizados, sendo portanto a função de auto-correlação entre o sinal identificado e a sua réplica calculada e analisada em alguns pontos, os quais são definidos de maneira arbitrária em relação à referência do receptor. Tais pontos não possuem relação com o sinal sendo transmitido pelo satélite, e servem para definir se o processo de correlação deve continuar, ou se a sincronização já foi atingida.

O presente trabalho se propõe a tornar a escolha destes pontos na etapa de correlação entre os sinais um processo determinístico, deixando assim de ser feito de maneira arbitrária. A mudança no modo de escolha dos pontos é introduzida pela UT, a qual por meio dos pontos sigmas calculados a partir da funçao densidade de probabilidade estimada do sinal, traz consigo informações e características da distribuição real dele.

Comentários e Conclusões dos Resultados Alcançados

Inicialmente, a discretização das funções de correlação e de distribuição de probabilidade é executada supondo-se que os sinais usados na simulação seguem a distribuição Normal padrão. Dos resultados obtidos, como possível visualizar na Figura 1.2, entende-se que tal suposição não se aplica a cenários que apresentam sinais de multipercurso. Estes, por apresentarem múltiplos picos em suas funções de correlação, não podem ser representados por uma distribuição Normal. Dessa forma, ao assumir os sigmas da distribuição Normal Padrão, assumi-se também que no local não há sinais de multipercurso.

Uma sugestão para o apontado no parágrafo anterior consiste em aplicar o mesmo procedimento de discretização da função correlação do sinal levando em consideração a sua própria distribuição de probabilidade, como visto na Figura 1.3. Dessa forma, a discretização apresenta-se com os pontos sigmas originais da distribuição do sinal, a qual possui as características reais do cenário em que o receptor se encontra e o percurso feito pelo sinal até ser recebido e processado.

Uma vez que se tem a função correlação do sinal na sua forma discreta, é possível aplicar uma segunda vez a UT de maneira a se obter um olhar mais próximo sobre o intervalo que apresenta a maior probabilidade de se encontrar o atraso da fase do código do sinal recebido. Dessa forma, da primeira aplicação da UT, tem-se a indicação na curva da função discreta, da região de maior probabilidade de se encontrar o arranjo ideal que permitirá a sincronização com o sinal. O intervalo identificado é separado e o restante da função descartado. A esse intervalo, que contém a maior probabilidade de máximo correlação, é aplicada a UT pela segunda vez, obtendo-se um olhar mais próximo da função correlação como visto na Figura 1.4.

A partir da segunda aplicação da UT, tem-se um olhar mais acurado e informações contidas no código do sinal podem ser repassadas ao usuário com maior grau de exatidão. Essa parte do

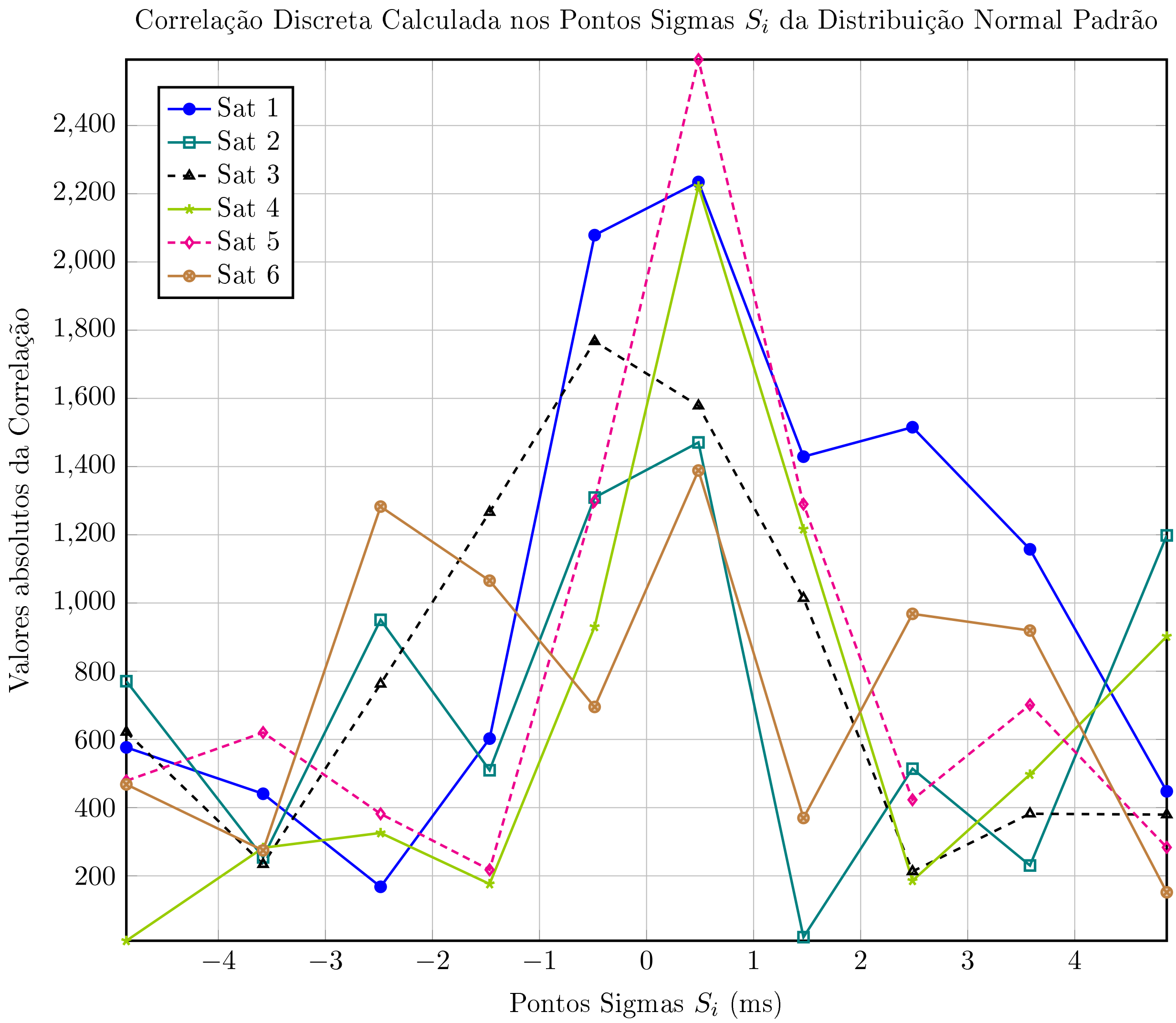

Figura 1.2: Funções correlação de seis sinais de satélite GPS realizadas nos pontos sigmas da UT para a uma função de densidade de probabilidade Normal Padrão.

trabalho também mostra que é melhor aplicar uma série de UTs seguidas, do que aplicar uma única UT com muitos pontos.

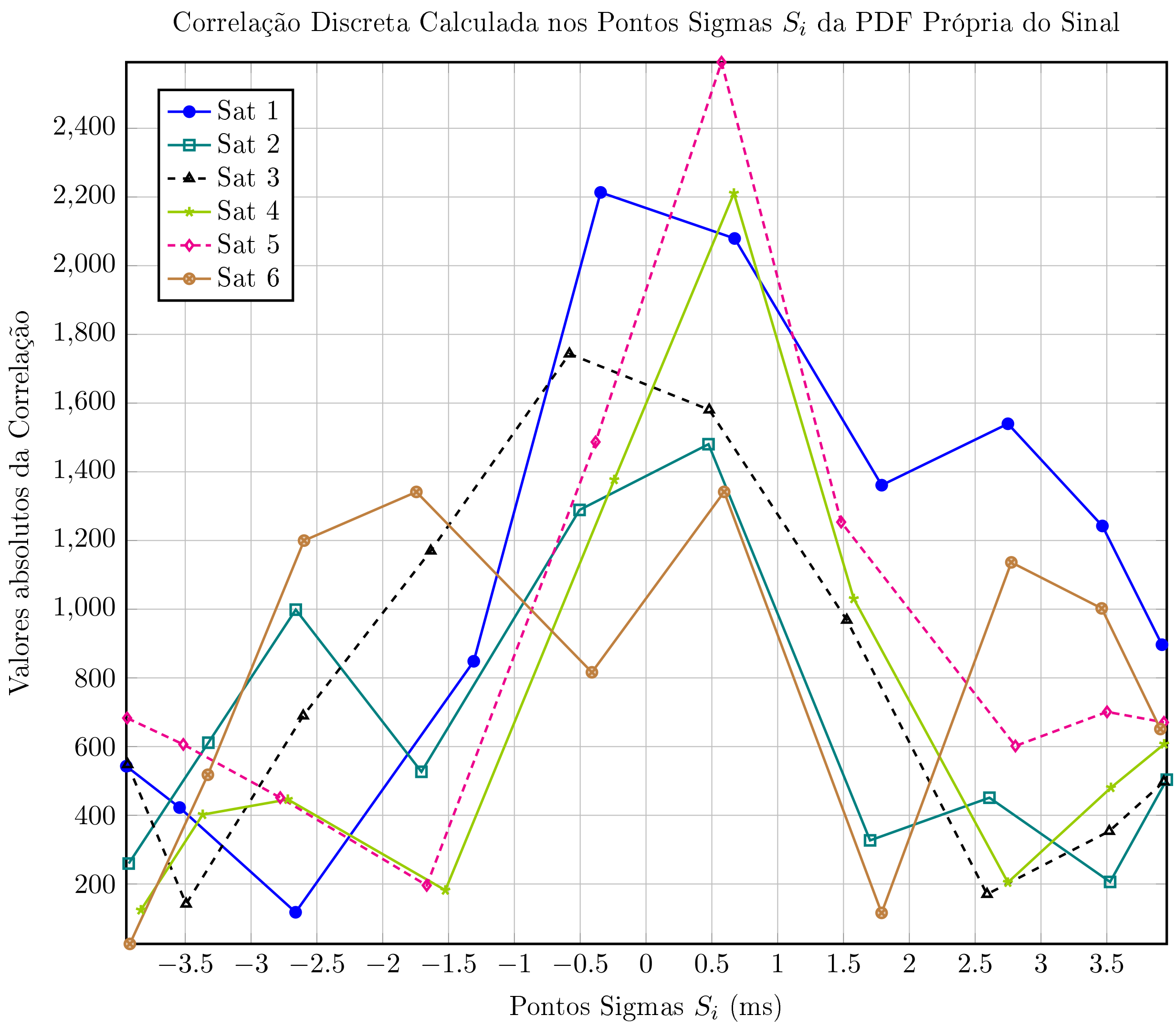

Figura 1.3: Funções correlação de seis sinais de satélite GPS realizadas nos pontos sigmas da UT com respeito à distribuição de probabilidade de cada sinal separadamente.

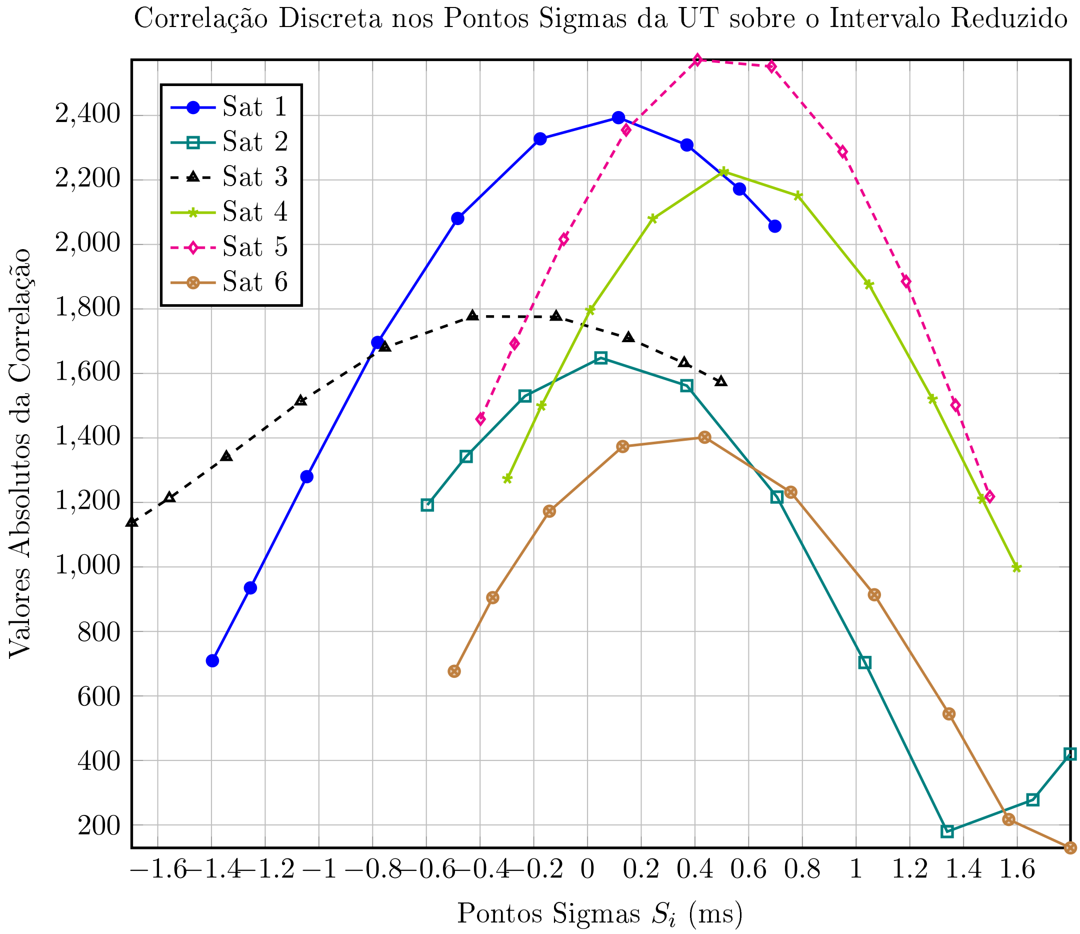

Figura 1.4: Discretização através da UT das funções de correlação reduzidas a seus intervalos de pico da função.

2 Introduction

Numerous estimation problems are typical of satellites and navigation, such as the measurement of time delay and direction of arrival, which provides knowledge of the mutual distance and angle coordinates between a transmitter and a receiver.

A GNSS receiver operates by measuring its distance from a group of satellites in space. One of the GNSS systems, the Global Positioning System (GPS) – developed and controlled by the United States – is committed to having at least 24 operational satellites, 95 % of the time. The use of a constellation of at least 24 satellites, which are located in six orbital planes at a height of 22 200 km, enables a GPS receiver to determine its location, speed, and direction anywhere on earth.

In order to calculate the distance between the receiver on the ground and the satellite, it is necessary to estimate with which delay the signal originally sent has reached the antenna, estimation known as Time-Difference of Arrival (TDOA). Basically, each satellite generates its signal in accordance with a clock on board. Then, the receiver generates a replica of the collected signal with respect to its own clock and attempts to align it with the incoming code by sliding the replica in time and computing the correlation. From the correlation property of the signal, the correlation function exhibits a sharp peak when the code replica is aligned with the code received from the satellite. That is when only one signal is involved. In the presence of noise and signals coming from multipath, the peak does not assume an acute shape.

Each GPS satellite continuously broadcasts its message via Binary Phase-Shift Keying (BPSK) at the rate of 50 bit/s. This message is sent through two spreading codes: one for the precise (P) mode (reserved for military use) and one for the Coarse/Acquisition (C/A) code. The C/A spreading code is a Pseudo-Random Noise (PRN) sequence with period of 1023 chips sent at 1.023 Mchips/s. The energy of the signal for civil users carrying a C/A-code is spread mainly over 2 MHz-wide frequency band, whereas the bandwidths of the signals for military users carrying a P-code are ten times wider.

The use of spread spectrum in the GPS system keeps the broadcasted signal resistant to jammings (external interference), since it has a broader spectrum than necessary. Also, spreading the signals spectrum brings a low signal power, which difficults the detection and interference in the signal. Another visible reason for using spread spectrum in the GPS system is that each satellite can use the same frequency band, allowing multiple transmitters, each given a distinct PRN spreading code, to transmit over the same spectrum simultaneously. Such multiple-access system is called Code Division Multiple Access (CDMA).

In this work the GNSS signal was modeled and simulated in the Matlab software. The model enables a closer approach to a GPS signal generation and its variables, such as the Satellite-Vehicle Identification Number (SVID), which provides the C/A-code being transmitted.

The signals transmitted by the GPS satellite system are also Right Hand Circular Polarization (RHCP).This polarization is simpler in terms of atenuating the effects provoked by the medium through which the signals travel from the satellites to a receiver, such as the Ionospheric Delay. Also, it allows the receiver to have any orientation without suffering any kind of interference.

2.1 Purpose of the Work

The current work aims to apply the UT at the estimation of the TDOA. Without using the UT, the TDOA is already estimated by correlating the incoming signal from the satellite with the local replicas generated at the receiver. Normally, the correlation function of the signal is discretized, and therefore considered only on some specific, previously aleatory decided, points. Instead of making the process always on those same points, in this work a discretization using the UT is proposed, which supported by its algorithm discretizes the correlation function on defined points. These points preserve the statistical moments of the distribution.

Besides being proposed fairly recently, the UT has been utilized in a variety of fields withing electrical engineering and is a powerful tool to transform a Probability Density Function (PDF) into a discrete version of itself, while preserving the moments of the distribution. Basically, the goal of the UT is to approximate the nonlinear mapping of a continuous Random Variable (RV) by the mapping of a set of deterministically selected points (later known as sigma points). Therefore, all estatistical moments are available by weights of the mapped values at the sigma points.

In this work, the application of the UT comes when calculating the correlation. It permits possible improvements in the process of estimating time delay, absolutely important in the communication and navigation of satellites. A deeper explanation on this mathematical transformation can be found in Chapter 5. Before, in Chapter 3, the necessary background on the probability theory, as well as, on the operations of a satellite receiver is presented. In Chapter 4 the description of the Data Model used on this work is given. The application of the UT for the purpose of this work comes in Chapter 6 and in Chapter 7. The obtained results is the subject of Chapter 8. A conclusion and the future perspectives related to this work are described in Chapter 9.

3 Fundamentals

3.1 Probability Theory

Before starting the future discussions about the UT, some basics elements and important terms of the theory of probability shall be reviewed [7] [3].

3.1.1 Random Variables

In probability and statistics a RV is a variable whose value is given by a random process following a certain probability distribution. These values are the outcome of an experiment (term used in probability theory to describe a process whose result cannot be totally predicted, because the conditions related to its performance can neither be predetermined with enough accuracy and completeness). Such experiment can give values that might represent the possible results in a future experiment, or are concerned to possible outcomes in past experiments, whose already existing values present uncertainties, due to imprecise measurement, for example.

If a random variable can only assume values belonging to a countable set, it is called discrete. For a discrete RV $\hat{x}$ the probability of $\hat{x} = x_i$ is given by $P_{\hat{x}}(x_i)$ that

$$\sum_i P_{\hat{x}}(x_i) = 1 \tag{3.1}$$

Furthermore, the probability distribution, mathematical function describing the possible values of a random variable and their associated probabilities, for a discrete RV is the Cumulative Distribution Function (CDF), where $F_x(x_i)$ of a RV $\hat{x}$ is the probability that $\hat{x}$ takes a value less than or equal to x_i such as

$$F_{\hat{x}}(x_i) = P(\hat{x} \leq x_i) \tag{3.2}$$

When a RV assumes any numerical values over an interval it is called as continuous random variable. In the continuity of any range, exists an uncountable number of possible values, and $P_{\hat{x}}(x_i)$, the probability that $\hat{x} = x_i$, as one of the uncountably infinite values, is always zero. The meaning for a continuous RV is on the probability that $x_i < \hat{x} \leq x_i + \Delta\hat{x}$, not on the probability $\hat{x} = x_i$.

The PDF defines a probability distribution for a continuous random variable. It follows that the probability of observing the RV $\hat{x}$ in the interval $(x_i, x_i + \Delta x)$ is $p_{\hat{x}}(x_i)\Delta x$ $(\Delta x \to 0)$. Thus, the probability of observing $\hat{x}$ in any interval is given by the area under the PDF $p_{\hat{x}}(x_i)$ over the interval Δx.

Every PDF must satisfy the following condition

$$\int_{-\infty}^{\infty} p_{\hat{x}}(x_i) dx_i = 1 \tag{3.3}$$

which represents the probability of observing $\hat{x}$ in the interval $(-\infty, \infty)$. It is also obvious that the PDF must not be negative, that is,

$$p_{\hat{x}}(x_i) \geq 0, \forall x \tag{3.4}$$

3.1.2 Statistical Moments

Since the concept of moments is extremely important in the discussion of RVs, a revision is made in this subsection.

Consider first the average value of a RV $\hat{x}$, which can also be called as mean, or expected value, and denoted as $E[\hat{x}]$. Let $\hat{x}$ assume n values $x_1, x_2, \ldots, x_n$. If there are N_i samples of $\hat{x}$, then the average $\overline{\hat{x}}$ of the RV $\hat{x}$ is given by

$$\overline{\hat{x}} = \sum_{i=1}^{n} x_i P_{\hat{x}}(x_i) \tag{3.5}$$

For a continuous RV $\hat{x}$ the expected value is similar to the discrete one and yelds

$$\bar{\hat{x}} = E[\hat{x}] = \int_{-\infty}^{\infty} \hat{x} p_{\hat{x}}(x_i) dx_i \tag{3.6}$$

The mean tells where a density is centered but gives no information about how spread out it is. The spread is measured by the Standard Deviation (SD) σ_x, which is the square root of another moment, the variance, denoted by σ_{x}^2. By definition,

$$\sigma_x^2 = E[(x - \bar{x})^2] \tag{3.7}$$

Thus, from Equation 3.7 the variance of x is equal to the mean square value minus the square of the mean. When the mean is zero, the variance is the mean square.

The nth moment of a RV $\hat{x}$ is defined as the mean value of $\hat{x}^n$. Thus, the nth moment of $\hat{x}$ is

$$\overline{\hat{x}^n} = \int_{-\infty}^{\infty} \hat{x}^n p_{\hat{x}}(x_i) dx_i \tag{3.8}$$

The nth central moment of a RV $\hat{x}$ is defined as

$$\overline{(\hat{x}-\bar{\hat{x}})^n} = \int_{-\infty}^{\infty} (\hat{x}-\bar{\hat{x}})^n p_{\hat{x}}(x_i) dx_i \quad (3.9)$$

3.1.3 Correlation

This section has been summarizing probability-theory, that is of special importance in the studies of random signals which might be encountered in communication problems. This subsection introduces the definition of correlation, which represents one way of describing the probabilistic relationship that can exist between two random variables like x and y.

The autocorrelation function $\psi_f(\tau)$ of a real signal $f(t)$ is defined as:

$$\psi_f(\tau) = \int_{-\infty}^{\infty} f(t)f(t-\tau)dt \quad (3.10)$$

The autocorrelation has its application in spread spectrum signals, which need to be demodulated. The demodulation of the spectrum-spreading modulation is accomplished by multiplying the received signal with a local reference of the spread code, identical in structure and synchronized in time with the received signal. That is the reason why autocorrelation is of great interest in choosing code sequences that give the least probability of a false synchronization.

Cross-correlation, which also measures the similarity between two different RV experiments, is of interest in several areas such as in CDMA systems, and in anti-jamming systems which must employ codes with low cross-correlation as well as unambiguous autocorrelation. The only difference between auto- and cross-correlation is that in the general convolution integral for autocorrelation in Equation 3.10, the function is not multiplied by a shifted version of itself, rather by another function:

$$\psi(\tau)_{(cross)} = \int_{-\infty}^{\infty} f(t)g(t-\tau)dt \quad (3.11)$$

3.2 Satellite Signal Acquisition and Tracking

As seen in Subsection 3.1.3, the correlation is an essential concept for GNSS receivers to synchronize with the incoming signal. They correlate first to identify which satellite signal has been received and, once the signal acquisition has been successful, the tracking mode starts. In this stage, the correlators are responsible for correlating the local generated replicas of the identified code with the incoming one by sliding the copied signals in time.

Hence, in order to achieve a better understanding of this work, it becomes imperative to comprehend the basics of a GNSS receiver, as well as, discern some important definitions and different processes involved. This section discusses in a condensed form about the topic [6], [7].

3.2.1 Digital Signal Processing

A digital communication system consists of several components. For instance, the radio frequency signal sent by the GPS is processed in a set of steps, handling the input signal from its digitalization and down-conversion, up to the GNSS receiver output.

As the incoming signal is received at the receiver antenna, it is leaded to the baseband processing block, where there are channels assigned to each SVID. Nowadays, the receivers are designed to process from 10 to hundreds of signals, that means they can support different number of channels, accordingly to the desired solution performance. Internally to each channel, the signal goes through other components – Doppler Removal, Correlators, Accumulators, Tracking Loops, Lock Detectors and Data Demodulation – that together makes the signal correlation and, its posterior baseband processing to the navigation possible.

3.2.2 Receiver Operations

In practice, a GNSS receiver must first replicate the PRN code that is transmitted by the satellite being acquired by the receiver; then it must shift the phase of the replica code until it correlates with the satellite PRN code. When cross-correlating the transmitted PRN code with a replica code, the same correlation properties arise that occurs for the mathematical autocorrelation process for a given PRN code. The mechanics of the receiver correlation process are very different from the autocorrelation process because only selected points of the correlation envelope are found and examined by the receiver.

When the phase of the GNSS receiver replica code matches the phase of the incoming satellite vehicle code, there is maximum correlation. When the opposite happens and the phase of the replica code is offset by more than 1 chip on either side of the incoming code, there is a minimum correlation. This is indeed the way in which a GNSS receiver detects the satellite signal when acquiring or tracking the incoming signal in the code phase dimension.

A typical GNSS receiver controls independent channels that are assigned to a specific signal from a specific SVID. Each one of this channels can operate in two modes: acquisition or tracking. In the acquisition mode, each channel searches in a two-dimension inspection (code delay and Doppler frequency) for the signal in order to know if the signal is present. This situation may be better visualized in Figure 3.1, where it is possible to see the acquired peak when the correlation assumes its maximum value. Until a signal is detected and a first estimate of the code delay and Doppler frequency is computed, the channel remains in acquisition mode. Once the estimates of the code delay and the Doppler frequency are continuously refined and, the navigation message is decoded, the channel changes into the tracking mode. In this mode, the channel controls if the signal is still present and also if the quality of the tracking results are correct. If the receiver loses the signal, the channel goes back to acquisition mode and restart the whole process. The Figure 3.2 gives an overview of the described operation. At the beginning, after start-up procedures (loading necessary data, initializations and resource allocations) the antenna starts providing digitized data continuously to the signal processing channels. The outputs of the channels (measurements and

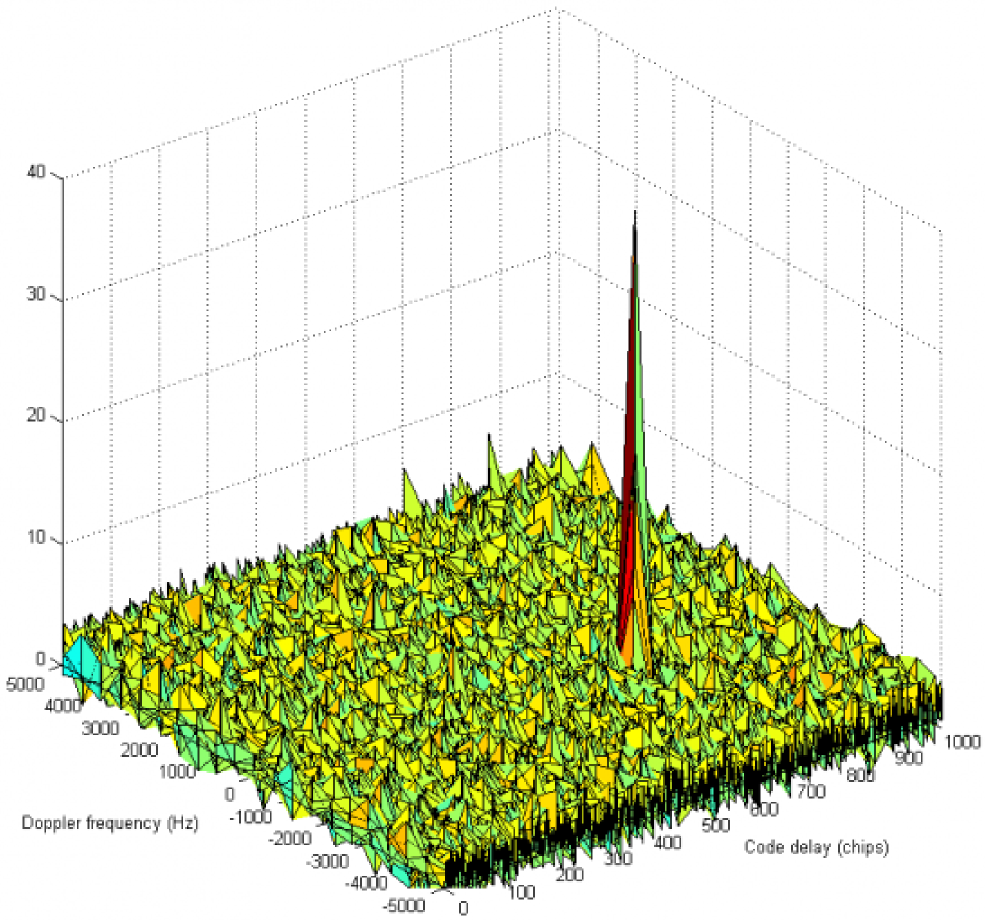

Figure 3.1: Example of an acquisition process, showing the Doppler/code delay search space and correlation peak that indicates presence of a signal with the code delay of 650 chips and Doppler frequency of -1.750 Hz. [10]

navigation message) are processed into usable solutions – 2D and/or 3D positioning, for example.

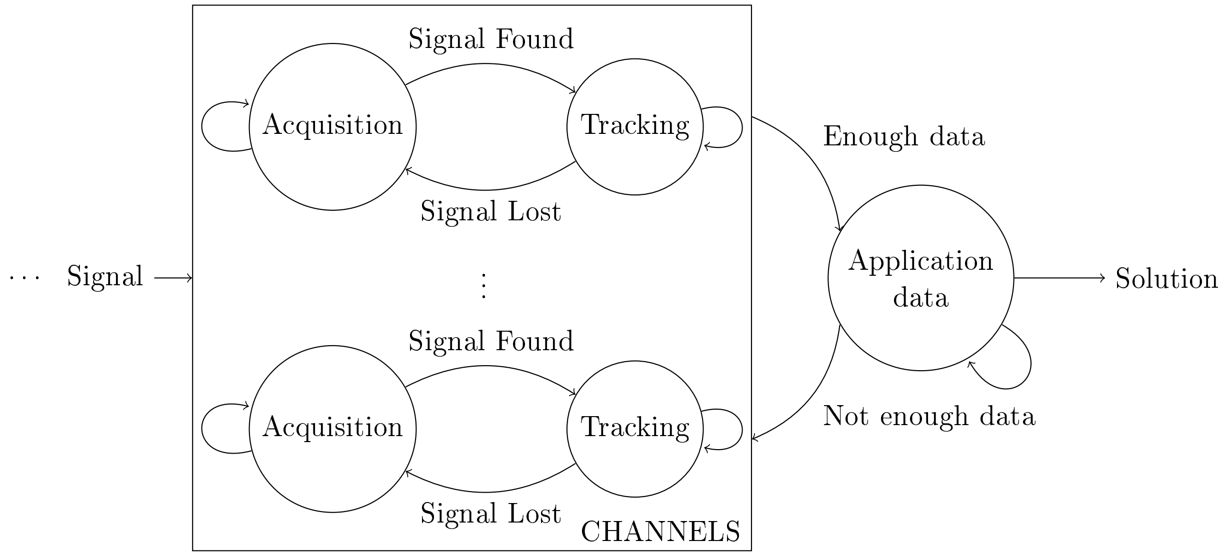

Figure 3.2: GNSS receiver operations diagram. A signal reaches the antenna and, after being processed, a solution is given accordingly to the desired purposes of the appliance.

4 Data Model

4.1 Notation

Throughout this work matrices will be denoted with uppercase upright bold letters $\mathbf{X}$, vectors will be denoted with lowercase upright bold letters $\mathbf{x}$, variables will be denoted with lowercase italic letters x, the value of the element at the index k of a vector $\mathbf{x}$ will be denoted by $\mathbf{x}[k]$.

4.2 Data Model

For this work, it was assumed that L narrow band waves are received by a system composed of only one antenna, as schema in Figure 4.1. In order to mathematically describe part of the data model used in this work, let a received signal with respect to one satellite, at a time instant t, be expressed as:

$$x(t) = s(t) + n(t) = \sum_{l=1}^{d} s_l(t) + n(t) \tag{4.1}$$

where $s(t) \, \epsilon \, \mathbb{C}^{L \times 1}$ stands for the superimposed signal replicas

$$s_l(t) = \gamma_l e^{j2\pi\nu_l t} c(t - \tau_l) \tag{4.2}$$

where, γ_l is the complex amplitude, ν_l is the Doppler frequency, and $c(t - \tau_l)$ denotes a PRN code sequence $c(t)$ that is periodically repeated with delay τ_l, chip duration T_c and period $T = N_c T_c$, with $N_c \, \epsilon \, \mathbb{N}$ representing the chip numbers. The noise present at time instant t is represented by $n(t)$. It was assumed the Additive White Gaussian Noise (AWGN), which follows the normal distribution $\mathcal{N}(0,\sigma^2)$ with zero mean and variance σ^2. The parameters of the LOS signals are indicated with $l = 1$ and the parameters of NLOS signals, i.e, the ones originated from Multipaths, with $l = 2, \ldots, L$. Since each satellite is identified by its unique PRN sequence, it was not considered necessary to define $s_l(t)$ in terms of the satellite. Multiple samples of the signal are collected at N different time instants.

The signal can be expressed in matrix notation as

$$\mathbf{y} = \mathbf{s} + \mathbf{n} = \boldsymbol{\gamma}(\mathbf{C} \odot \mathbf{D}) + \mathbf{n} \, \epsilon \, \mathbb{C}^{1 \times N} \tag{4.3}$$

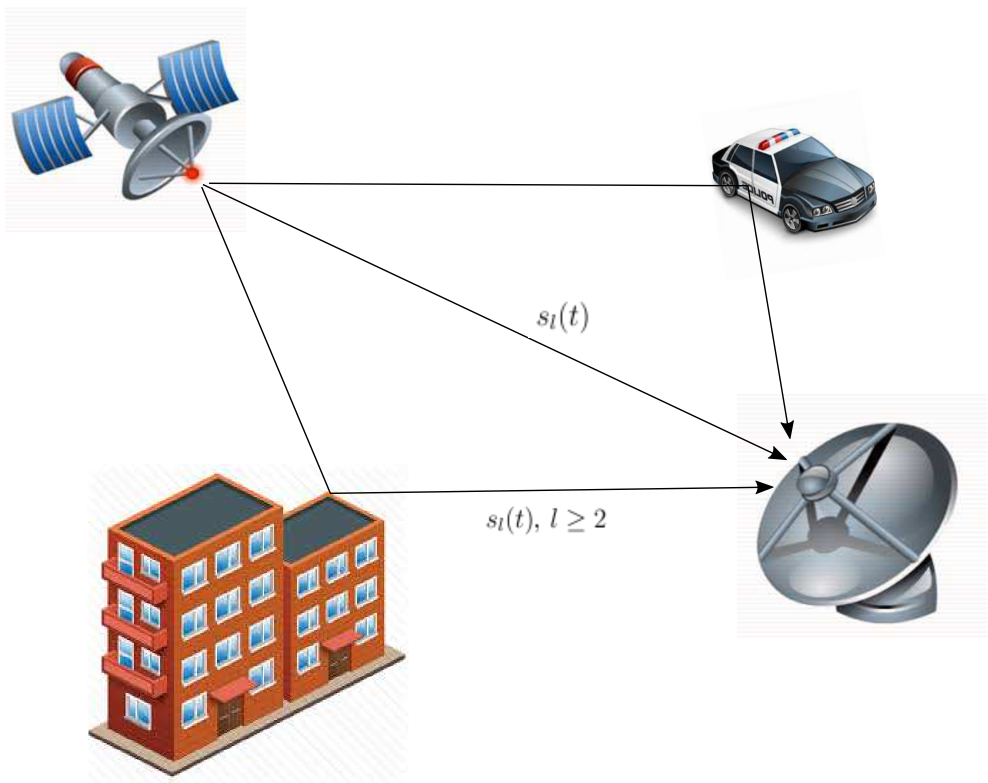

Figure 4.1: Schema of the Data Model used on this work. Only one antenna receive LOS and NLOS signals. (Icons on the Figure taken from: iconfinder.com; iconexperience.com and findicons.com).

where $\odot$ represents the Hadamard-Schur product,

$$\boldsymbol{\gamma}= [\gamma_1, \ldots, \gamma_l, \ldots, \gamma_L]^{\mathrm{T}} \,\epsilon\, \mathbb{C}^{\mathbb{L}\times 1}$$

is a vector that contains the complex amplitudes of the signal replicas

$$\mathbf{C} = [\mathbf{c}\,[\tau_1] \ldots \mathbf{c}\,[\tau_l] \ldots \mathbf{c}\,[\tau_L]]^{\mathrm{T}} \;\epsilon\; \mathbb{C}^{L\times N} \tag{4.4}$$

has the sampled and shifted $c\,(t)$ for each wavefront that reaches the antenna receptor,

$$\mathbf{D} = [\mathbf{d}\,[\nu_1] \ldots \mathbf{d}\,[\nu_l] \ldots \mathbf{d}\,[\nu_L]]^{\mathrm{T}} \;\epsilon\; \mathbb{C}^{L\times N} \tag{4.5}$$

contains the complex exponential functions which conveys to the Doppler frequency of each wavefront.

GNSS signals apply the signaling concept of CDMA. Therefore it is possible that many satellites transmit their signals on the same frequency without losing data. The received signal $\mathbf{y}$ is seen as the sum of others signals, composed of the signal from the satellite, as well as noise and delayed replicas arriving due to the multipath.

For the purpose of this work, local replicas $\mathbf{r}\epsilon\, \mathbb{C}^{\mathrm{m}\times 1}$ of the signals from each satellite of the GPS system are generated with different $m \,\epsilon\, \mathbb{N}$ delays α

$$\mathbf{r}[\alpha] = e^{j2\pi\nu_l t} c\,(t - \tau_\alpha) \tag{4.6}$$

and a correlation with the sum of arrived signals $\mathbf{y}$ has benn calculated:

$$\mathbf{Z} = \mathbf{r}\mathbf{y} \,\epsilon\, \mathbb{C}^{m\times N} \tag{4.7}$$

Since the correlation function describes the closeness or resemblance between signals, it helps to find out the shifted signals that most approach to the one received.

5 Unscented Transform

The UT was developed by Julier and Uhlman in 1997 [4], [5]. In [4] Julier presented the UT as the “distribution approximation filter”. The mathematical theory of the UT is based on the estimation of the moments of an estatistical distribution. One possible way to perform this is to approximate the continuous distribution $w(\hat{u})$ of a RV $\hat{u}$ by a discrete distribution $w_i(\hat{u})$, existing only at discrete points S_i. The approximation is implemented so that the mapping of the two distributions yields the same moments after a nonlinear mapping.

Let the continuous distribution have its moments defined as:

$$E_c\left\{\hat{u}^k\right\} = \int\limits_{-\infty}^{\infty} \hat{u}^k w(\hat{u}) \tag{5.1}$$

Whereas the discrete distribution is defined only at selected points S_i, called as sigma points. Therefore the moments become:

$$E_d\left\{\hat{u}^k\right\} = \int\limits_{-\infty}^{\infty} \hat{u}^k w_i(\hat{u})\, d\hat{u} = \int\limits_{-\infty}^{\infty} \hat{u}^k \left[\sum_i w_i \delta(\hat{u} - S_i)\right] d\hat{u} = \sum_t w_i S_i^k \tag{5.2}$$

After that, a random variable has two distributions: a continuous and a discrete one.

The weights w_i and sigma points S_i are calculated from (5.1). Then the nonlinear mapping is represented as:

$$\sum_i w_i G(S_i) = \int\limits_{-\infty}^{\infty} G(\hat{u})\, w(\hat{u})\, d\hat{u} \tag{5.3}$$

Equation (5.3) shows that the UT expression for the expected value is an approximation of the integral of the nonlinear mapping function $G(\hat{u})$ weighted by a function $w(\hat{u})$. The nonlinear mapping stands for the process that will affect the RV $\hat{u}$. The process may be an analytical equation or a numerical computation.

Table 5.1, in the end of this current chapter, shows the weights and sigma points for the different probability density functions. The sigma points are always part of the original distribution. Both constitute a discrete set that satisfies all the requirements for a probability distribution.

Since the discrete probability function is an approximation of the continuous one, as the number

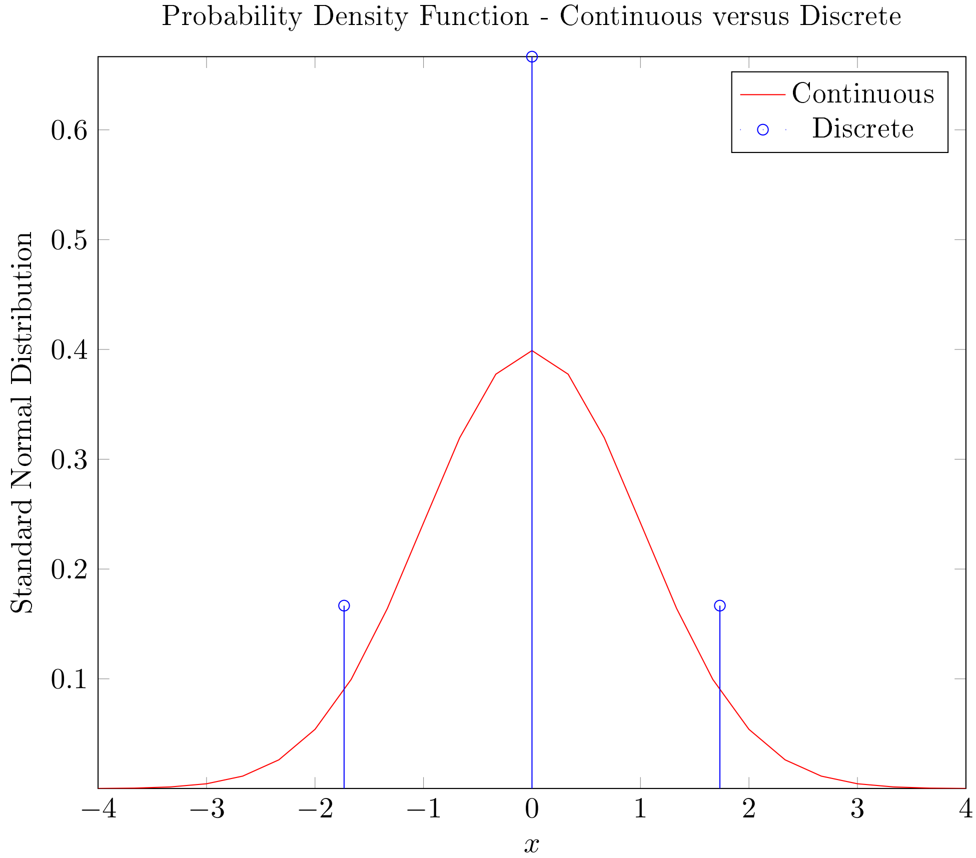

Figure 5.1: Representation of the continuous normal distribution and the discrete approximation.

of sigma points grows, the discrete function converges to the continuous one. Figure 5.1 shows the probability density function for the discrete (here, using three sigma points) and continuous normal distribution. The larger the number of sigma points, the closer the value of the midpoints (between sigma points) is to the continuous distribution function. The same can be applied in the uniform and exponential cases.

The UT can be formulated using the Taylor expansion of the nonlinear mapping [8], [1], [2]. Considering that $\hat{u}$ is a zero mean RV with known probability distribution, the Taylor series is another way to find suitable values of the sigma points S_i as well as the discrete probabilities w_i of the UT.

$$G\left(\overline{U}+\hat{u}\right)=G\left(\overline{U}\right)+\frac{dG}{du}\hat{u}+\frac{1}{2!}\frac{d^2G}{du^2}\hat{u}^2+\frac{1}{3!}\frac{d^3G}{du^3}\hat{u}^{3+\ldots} \tag{5.4}$$

The formulation representing the Taylor series (5.4) as a polynomial is more compact, therefore in order to simplify efforts it will be used from this point on:

$$G\left(\overline{U}+\hat{u}\right)=G\left(\overline{U}\right)+p\left(\hat{u}\right) \tag{5.5}$$

The expected value of (5.5) can be found as:

$$\overline{G} = E\left\{G\left(\overline{U} + \hat{u}\right)\right\} = E\left\{G\left(\overline{U}\right)\right\} + E\left\{p\left(\hat{u}\right)\right\} = G\left(\overline{U}\right) + \overline{P} \tag{5.6}$$

In (5.6), $\overline{P}$ is the expected value of the Taylor polynomial. The variance of (5.5) is:

$$\sigma_g^2 = E\left\{\left[G\left(\overline{U} + \hat{u}\right) - \overline{G}\right]^2\right\} = E\left\{p\left(\hat{u}\right)^2\right\} - \overline{P}^2 \tag{5.7}$$

Once (5.5) is also valid for (5.2), the Taylor representation is likewise usable for the sigma points.

$$G\left(\overline{U} + S_i\right) = G\left(\overline{U}\right) + p\left(S_i\right) \tag{5.8}$$

The polynomial remains the same, because the sigma points S_i belong to the probability distribution of $\hat{u}$. The comparison between (5.6), (5.2) and (5.8) results in the first set of conditions for the sigma points.

$$\left[w_0 + \sum_i w_i\right] \quad = \quad 1 \tag{5.9}$$

$$\sum_i w_i p\left(S_i\right) = E\left\{p\left(\hat{u}\right)\right\} = \overline{P} \tag{5.10}$$

The comparison of (5.7) with (5.2) and (5.8) results in the second set of conditions for the sigma points:

$$\sum_i w_i p\left(S_i\right)^2 = E\left\{p(\hat{u})^2\right\} \tag{5.11}$$

It is possible to combine the results from (5.9) to (5.11) in a single set of equations:

$$w_0 = 1 - \sum_i w_i \tag{5.12}$$

$$\sum_i w_i S_i^k = E\left\{\hat{u}^k\right\} \tag{5.13}$$

The order of approximation is k, which is given by how one truncates the polynomial.

Consequently, the truncation of the Taylor polynomial determines the number and value of the sigma points S_i as well as the weights w_i of the UT. A careful look at (5.13) shows that a second-order polynomial ($k = 2$) is the minimum practical option. This has an outcome of a nonlinear system with four equations and four variables (w_1, w_2, S_1, S_2). The equations for a zero mean probability distribution with skewness γ_1 and kurtosis γ_2 are:

$$\sum_{i=1}^{2} w_i S_i = E\left\{\hat{u}\right\} = 0 \tag{5.14}$$

$$\sum_{i=1}^{2} w_i S_i^2 = E\left\{\hat{u}^2\right\} = \sigma^2 \tag{5.15}$$

$$\sum_{i=1}^{2} w_i S_i^3 = E\left\{\hat{u}^3\right\} = \gamma_1 \sigma^3 \tag{5.16}$$

$$\sum_{i=1}^{2} w_i S_i^4 = E\left\{\hat{u}^4\right\} = \left(\gamma_2 + 3\right) \sigma^4 \tag{5.17}$$

The solution of these equations results in the weights and sigma points for any given probability distribution, provided the first central moments are finite and known.

$$S_1 = \sqrt{\frac{\gamma_2 + 3}{1 - \gamma_1 + \gamma_1^2}}\sigma \tag{5.18}$$

$$S_2 = -\left(1 - \gamma_1\right) \sqrt{\frac{\gamma_2 + 3}{1 - \gamma_1, + \gamma_1^2}}\sigma \tag{5.19}$$

$$w_1 = \frac{1 - \gamma_1 + \gamma_1^2}{6 + 2\gamma_2 - 3\gamma_1 - \gamma_1\gamma_2} \tag{5.20}$$

$$w_2 = \frac{1}{\left(1 - \gamma_1\right)} \frac{1 - \gamma_1 + \gamma_1^2}{6 + 2\gamma_2 - 3\gamma_1 - \gamma_1\gamma_2} \tag{5.21}$$

The reason why the sigma points approximate well the nonlinear mapping is connected to the Taylor series representation of the mapping (5.4). In fact, the sigma points are where the truncated mapping (5.5) is exactly calculated. This is true for any probability distribution, as long as all the moments exist. The order of the truncated polynomial determines the accuracy of the approximation. Higher order polynomials demand the knowledge of higher order moments.

N	Weights					Sigma Points					Function
2	0.5	0.5				−0.577	0.577				
3	0.278	0.444	0.278			−0.775	0	0.775			$w(\hat{u}) = \begin{cases} \frac{1}{2} & \lvert\hat{u}\rvert \leq 1 \\ 0 & \lvert\hat{u}\rvert > 1 \end{cases}$
4	0.174	0.326	0.326	0.174		−0.861	−0.340	0.340	0.861		
5	0.118	0.239	0.2845	0.239	0.118	−0.906	−0.538	0	0.538	0.906	Uniform
2	0.5	0.5				−0.707	0.707				
3	0.333	0.333	0.333			−0.866	0	0.866			$w(\hat{u}) = \begin{cases} \frac{1}{\pi * \sqrt{1-\hat{u}^2}} & \lvert\hat{u}\rvert \leq 1 \\ 0 & \lvert\hat{u}\rvert > 1 \end{cases}$
4	0.25	0.25	0.25	0.25		−0.9238	−0.3827	0.2827	0.9238		
5	0.2	0.2	0.2	0.2	0.2	−0.951	−0.5878	0	0.5878	0.951	
2	0.854	0.146				0.586	3.414				
3	0.711	0.279	0.010			0.416	2.294	6.290			$w(\hat{u}) = \begin{cases} e^{-\hat{u}} & \hat{u} > 0 \\ 0 & \hat{u} < 0 \end{cases}$
4	0.6031	0.3574	0.0389	0.0005		0.323	1.746	4.537	9.395		
5	0.5218	0.3987	0.0759	0.0036	0.000 02	0.264	1.413	3.596	7.086	12.641	Exponential
2	0.5	0.5				−1	−1				
3	0.167	0.666	0.167			−1.732	0	1.732			$w(\hat{u}) = \frac{1}{\sqrt{2}} e^{\frac{\hat{u}^2}{2}}$
4	0.107	0.337	0.337	0.107		−2.334	−0.742	0.742	2.334		
5	0.011	0.222	0.5333	0.222	0.011	−2.857	−1.356	0	1.356	2.857	Gaussian

Table 5.1: Weights, sigma points and corresponding Probability Density Functions. [2]

6 Accomplishing Discrete Functions through the Unscented Transform

The application of the UT on this current study is related to the signal delay estimation. Basically, the generated replica signals are slided in time with a sigma S_i delay. It means that the shift in time for the replica signals comes from the sigma points of the UT. By doing this and calculating the correlation between the delayed replica signals and the original one received, the result comes out with the most probable interval in which the proper delay is, and so the aimed synchronization.

In the next experiments, six signals from different satellites of the GPS system are simulated, where two of them are NLOS and so, they add multipath. In order to bring this work closer to the reality, AWGN is also considered. As a result, there is a sum of signals from six different satellites, presence of multipath-signals and noise, all of it arriving at the antenna receiver. A situation that can model effectively how the receiver literally sees the incoming signals: as a sum, that not only contains the important information, but also noise and signals from multipath.

Figure 6.1 shows how the sum of received signals can be changed by the presence of noise. Note that in this case, the Signal-to-Noise Ratio (SNR) used is of $-15\,\mathrm{dB}$, as an attempt to bring the simulation close to the reality. Satellite signals use the spread spectrum, therefore they have their signal spread over a vast bandwidth. Because of that, the received signal presents a low spectral density and is lower than the noise floor, as noticed from the negative SNR value.

In order to fulfill the purpose of this work and investigate the use of the UT, two distinct ways of estimating the TDOA have been used. Both of them apply the UT: the first one is presented in Section 6.2 and uses the sigmas from the Standard Normal Distribution and the second, in Section 6.3, handles the sigmas from the signal's own distribution function. Before, in Section 6.1, the correlation function is presented in its smoothly form, in order to serve as the benchmark for the outcomes of this work. To calculate the UT only the statistical moments of the distribution are necessary. In order to accomplish this requirement and find the probability density function of the signal, the correlation is first made on discrete points, which after having its points interpolated, presents continuous characteristic and can be manipulated to find an estimate of the PDF.

Note: to improved readability all the figures on this Chapter can be found at the end of it.

6.1 Interpolation to Obtain the Probability Density Function of the Signal

As an important step to start this work, the replicas of the incoming signal are generated with short time interval delays, which gives the smooth characteristic to the correlation curves and so to its PDF.

From Chapter 5, the only condition to apply the UT is the previous knowledge of the statistical moments of the function, making this mathematical transformation suitable for many probabilistic distributions. Thus, under any circumstances, the information of the signal's distribution has to be achieved so the application of the UT can actually happen. This work proposes interpolating the correlations done at the bank of correlators at the GPS receiver. Each correlator at a bank of correlators correlates the signal by applying a code phase delay, the correlation on these points are integrated over time and interpolated. From the obtained interpolated correlation curve, an estimate of the PDF can be obtained as well as estimates for its moments.

The curves in Figures 6.2 and 6.3 are the correlation function of the incoming signal and its probability density function, respectively. They both show from their peaks, where the applied delay most closely matches the one from the identified received signal.

Note that Figures 6.2 and 6.3 serve as the start point for the rest of this work. It is from them that the statistical information of the signals are taken off. By reducing the sample points through a discretization of the curves using the UT, the discrete curves should not necessarily look similar, since the UT's purpose is to preserve the statistical properties of the signal distribution function.

6.2 Applying the Unscented Transform Assuming the Standard Normal Distribution

This first application of the UT in this work comes from Figures 6.2 and 6.3. Because of their symmetric behavior around their means, it is possible to notice a resemblance between the obtained curves and one that follows the Standard Normal Distribution. The similarity provides a good reason to try an approximation using sigma points obtained from this distribution function. The results are the shown in Figures 6.4 and 6.5 with the signal's sample-based time delay being 0 ms, 0.1 ms, 0.2 ms, 0.3 ms, 0.4 ms and 0.5 ms, respectively for each of the six satellites.

Figures 6.4 and 6.5, respectively the discrete correlation curve and the signals' probability density function, present the discrete version of the continuous functions obtained in Section 6.1. The application of the UT here comes in a context in which all the six involved signals on the simulations would follow the Standard Normal distribution. Despite the fact that not all PDFs are centered around zero, the sigma points for the UT can be easily displaced to take into account the new mean. Furthermore, the receiver can set its reference at any point. Here the delays differ from each other in order to make the identification of the six different signals easier.

The applicability of the UT for the Standard Normal distribution showed not to be a good solution in scenarios with multipath. In the presence of NLOS signals, the real mean can be slightly distorted from the assumed reference and not assume a proper bell curve shape. Placing the receiver where multipath signals do not represent a problem, the UT from the Standard Normal distribution is a fairly good assumption. In this situation, the sigmas are already known and can be directly applied for generating the signal replicas. As will be presented in Section 6.3, the sigmas can also be individually calculated for each incoming signal, what can maybe take slightly more time than when considering the signals to follow the Standard Normal distribution, whose sigma points are already known.

6.3 Unscented Transform with Respect to the Real Function Distribution of the Signal

In this section, the sigma points from the UT were obtained from the calculation of the moments, which uses the PDF interpolated from each one of the six satellites. Figures 6.6 and 6.7 show the discrete estimation of the correlation and probability density curves, respectively. In this case, the discrete representation of those curves by calculating the UT uses the real moments of the distribution of the signal coming from each satellite, and not the moments of the Standard Normal distribution, as discussed in the previous section.

In order to get a better visualization of how close each discretization gets to the continuous case, a plot with the three situations is generated and put together on the same graphic, as shown in Figure 6.8, where has been represented the signal of only one satellite when no multipath is present.

From Figure 6.8, one cannot easily say which of the two approximations of the UT gets the closest to the smoothest curve of the correlation, if it is the one that assumes the moments from the Standard Normal Distribution, or if it is the one that specifically calculates the statistical moments of the unknown distribution. Since the statistics moments provide sufficient information to reconstruct a PDF, the first two moments of the distribution in each approximation has been calculated and then showed in Table 6.1 in order to comparison.

Kind of the curve	Mean	Variance
Continuous distribution	0.1601	5.8158
Standard Normal UT	0	1
Calculated Moments UT	0.1601	5.8158

Table 6.1: Comparison between the continuous case and its discretizations using the UT, through its estatistical moments.

From the results in Table 6.1 of the statistical moments it is pretty clear the conservation of the moments practiced by the UT. Calculating the moments by applying the sigmas and weights from the distribution gives the same values of the original distribution (here identified as Continuous distribution) when comparing with the Calculated Moments UT. The mean and variance values for the Standard Normal UT, as expected for this distribution, assume zero mean and unitary variance.

Undoubtedly, applying the sigmas and weights from the Standard Normal distribution was also a good way to estimate the moments. The similarity with the distribution comes not only in the curves, but also in the numbers, as it is possible to observe from Table 6.1. The mean in the original distribution assumes a value close to zero. Remember that time sample-based can be referenced to zero at the receiver for all the signals, here they have been just closed chosen, so one may identify the signals easier. The variance, however, increases and does not assume an unitary value, which means that the data points are spread out around the mean. Such result for the variance comes mostly from the noise and the presence of multipath signals.

Also as a result from the observation of the figures generated to this chapter, one can say that the UT that follows the Standard Normal distribution is not recommended in cases of strong multipath, since in such scenarios the incoming signal would mostly not assume a zero (or even close to zero) mean. Thus, the signal code-delay estimation would probably face inaccuracies and do not give proper solutions.

In fact, there is a trade-off, since both provide good results. For the one that applies the sigmas accordingly to the signal's own distribution function, the points can either be calculated at the moment the satellite signal reaches the receiver, or calculated before, taking different scenarios into consideration and have them saved at the receiver integrated system to be used when correlating the signals. The first option may come up with a more accurate time-shift delay estimation, since it is done individually for each situation at the instant the signal arrives. On the other hand, it may take longer than when generalizing the points for every situation. It depends on the desired application and what is most important for the user.

Note: The discontinuities presented in Figures 6.2, 6.3 and 6.8 are originated from the presence of noise, since a SNR of $-15\,\mathrm{dB}$ has been used, and such interruptions also represent the end of the code period during the time sampled. The domain of the curves are also in terms of T_s, the sample time, in $\frac{1}{2\,\mathrm{MHz}}$.

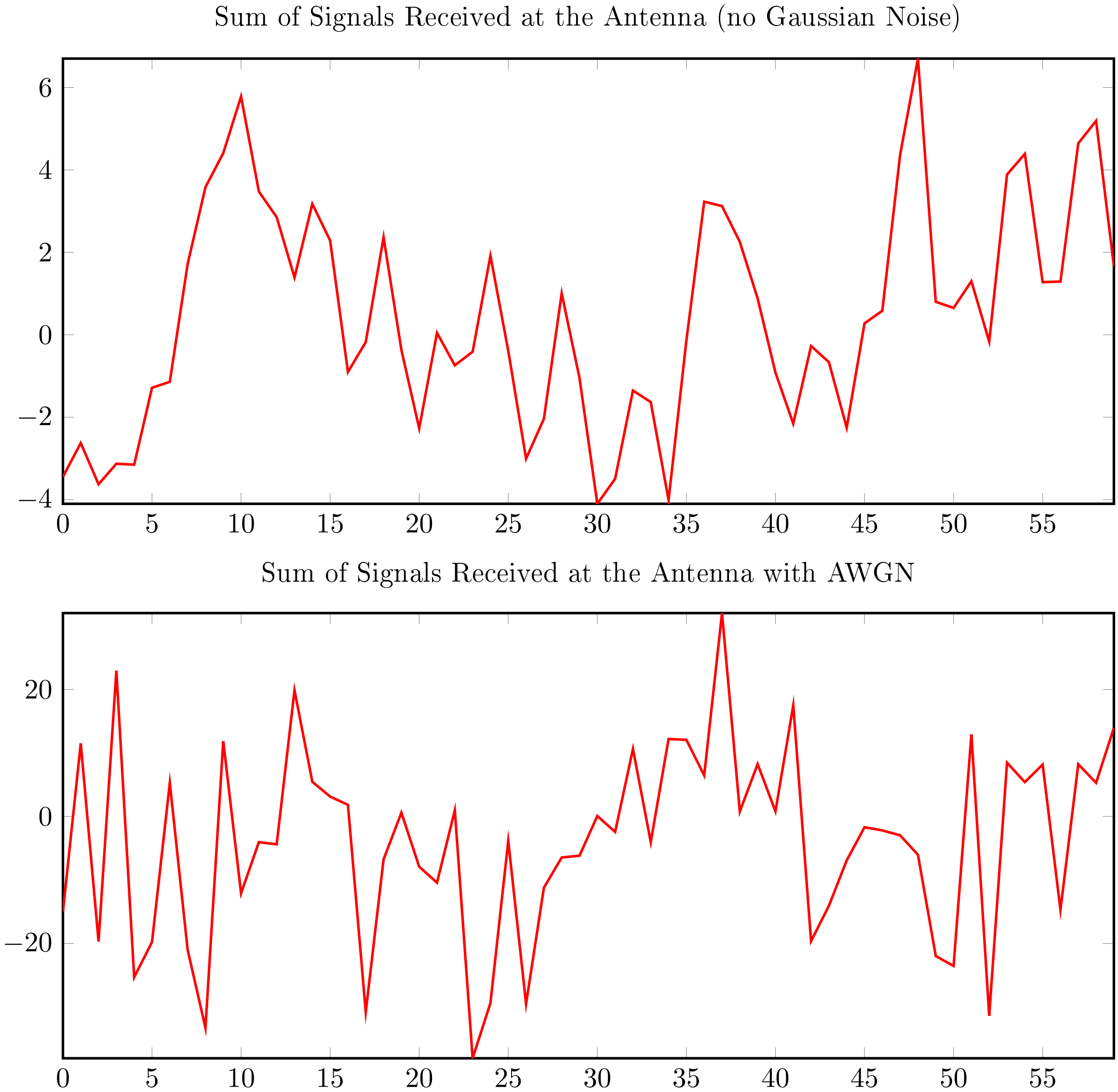

Figure 6.1: Overview of the sum of arrived signals with and without noise.

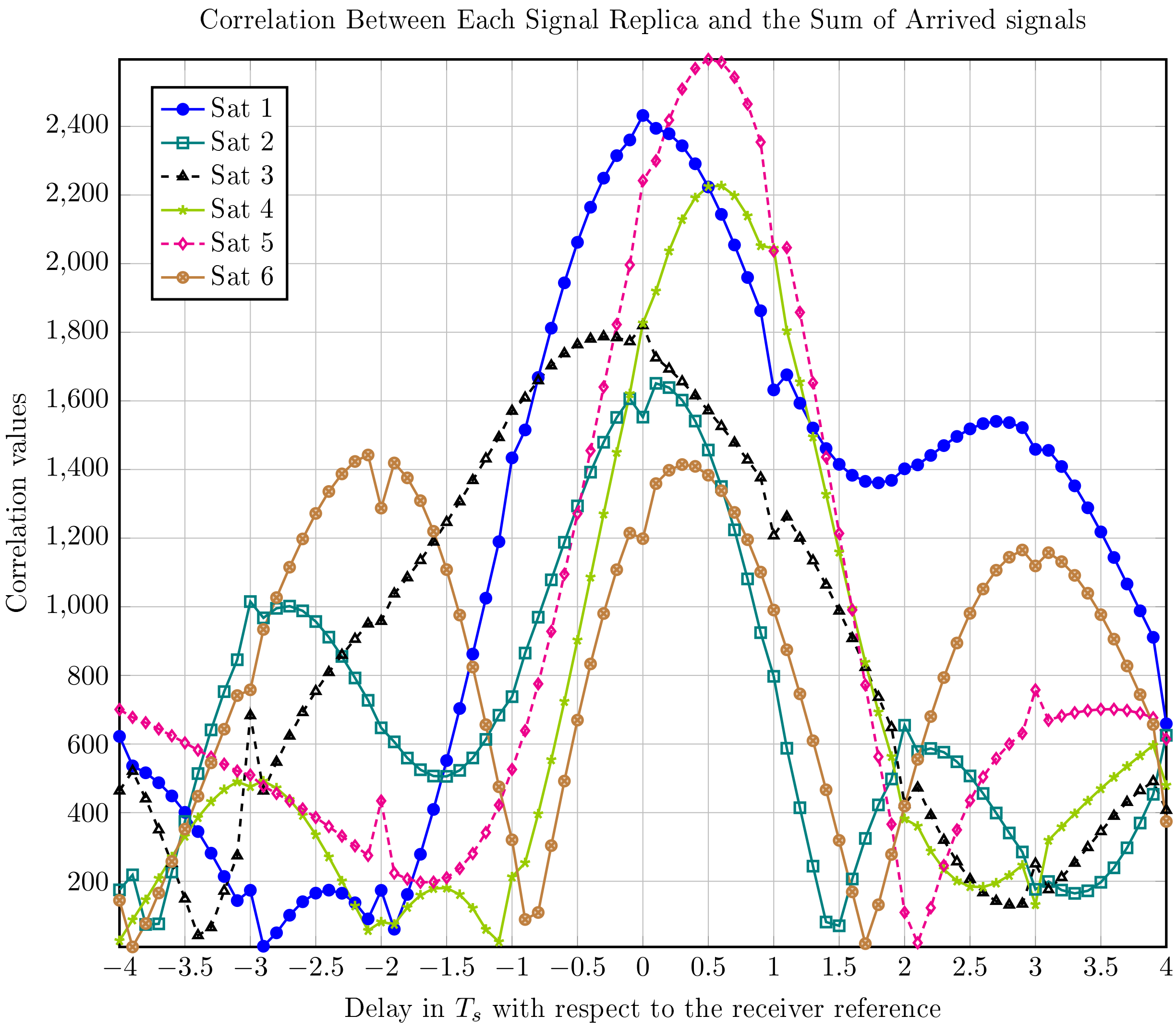

Figure 6.2: Correlation calculated from the almost continuous delays intervals applied to the generated replicas of the satellite signals.

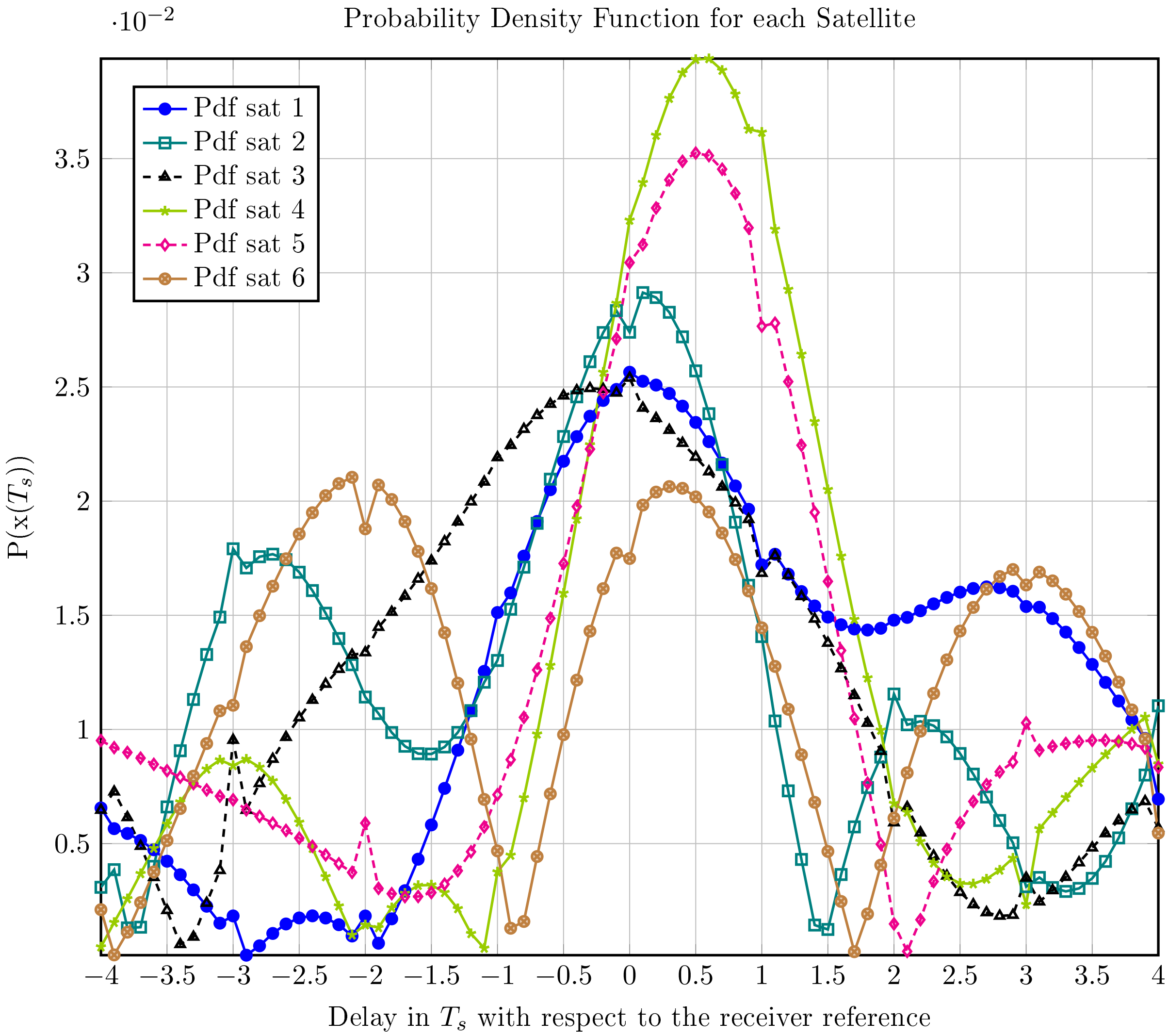

Figure 6.3: Probability Density Function for the six satellites in its continuous form.

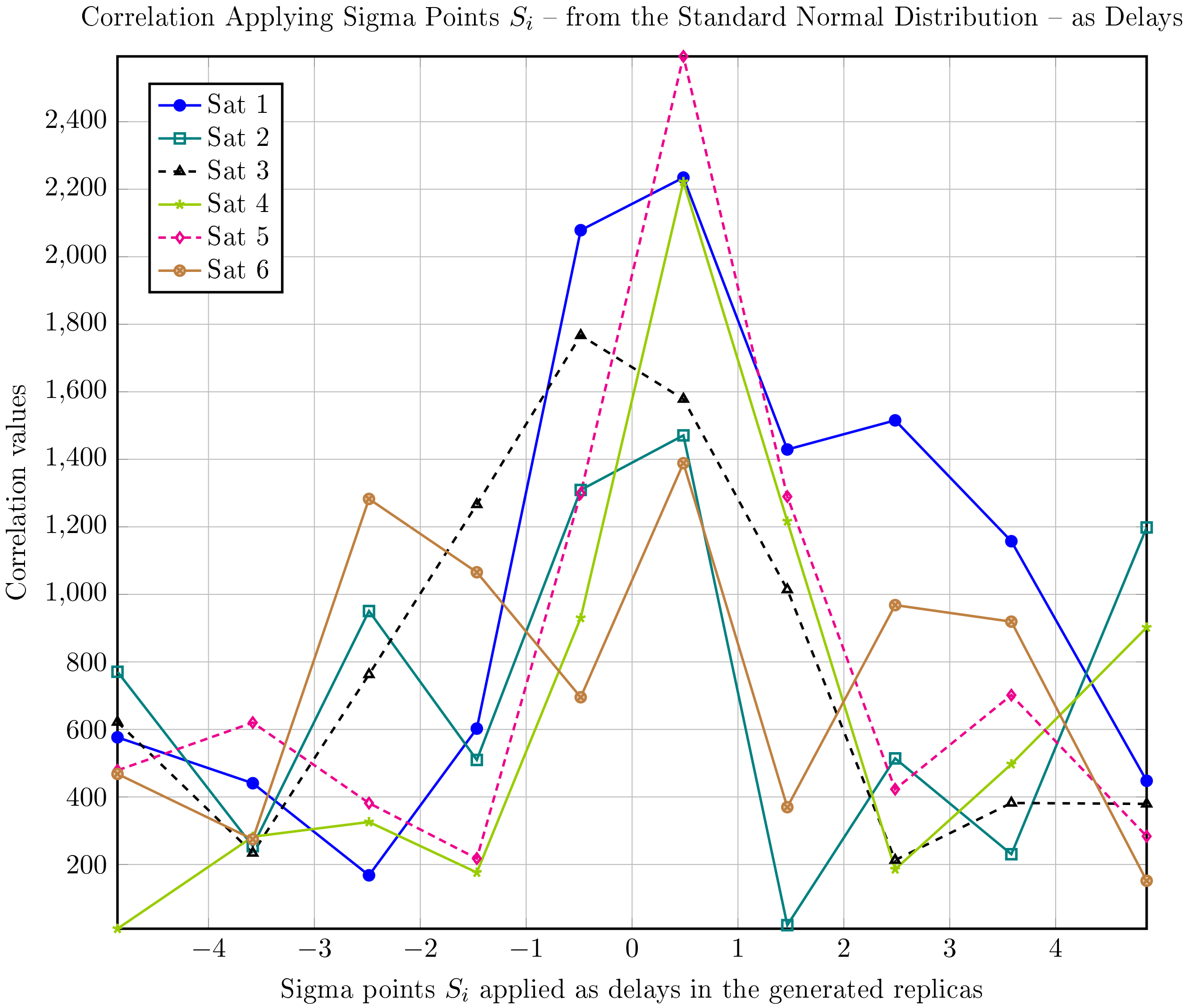

Figure 6.4: Approximation of the correlation curve using sigma points from the UT of a Standard Normal Distribution. Instead of correlating in many points as did in Figure 6.2, the sigmas are used as delays in the generated replicas at the receiver producing a discrete correlation curve.

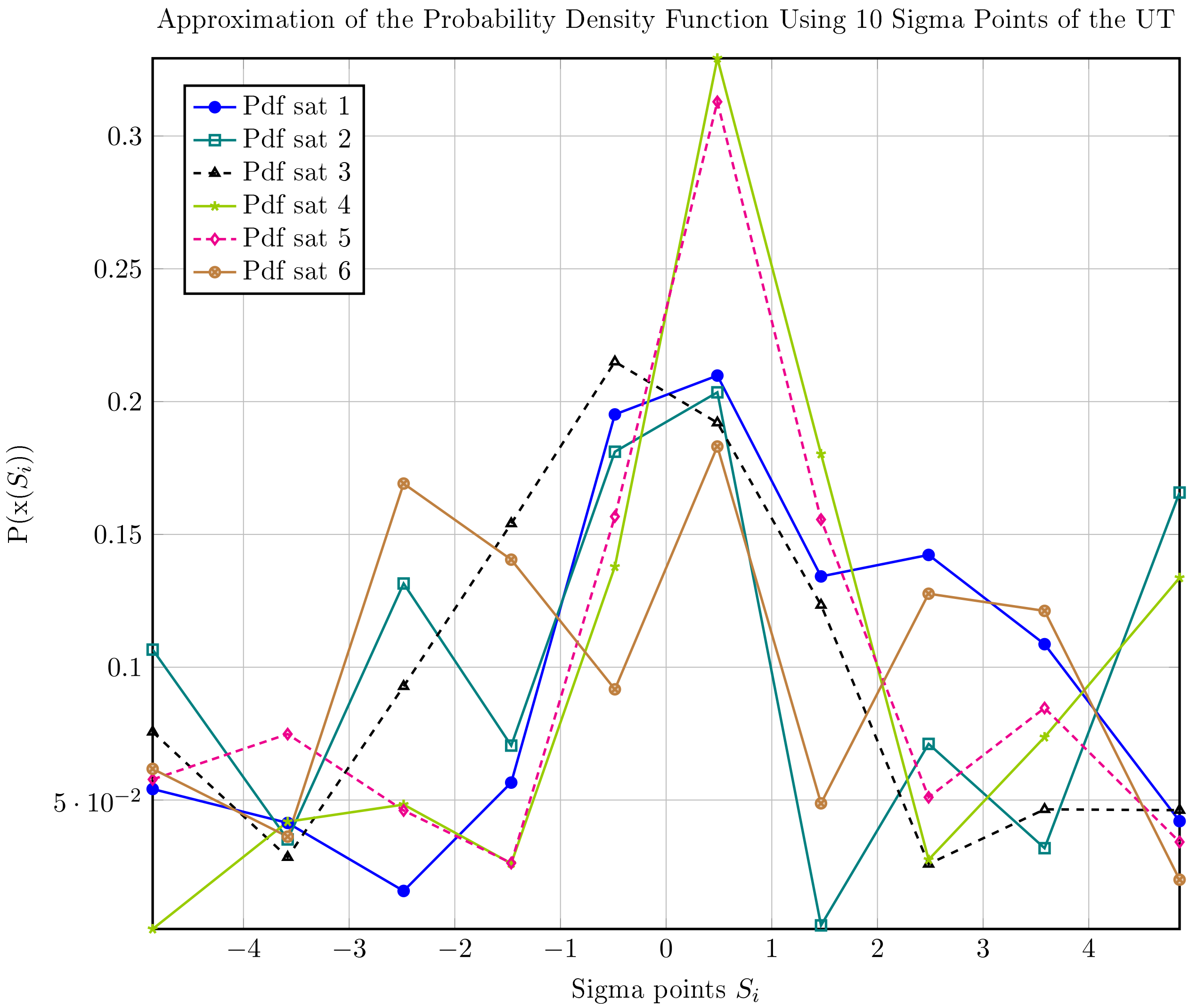

Figure 6.5: Probability Density Function calculated using the sigma points from the Unscented Transform that follows the Standard Normal Distribution.

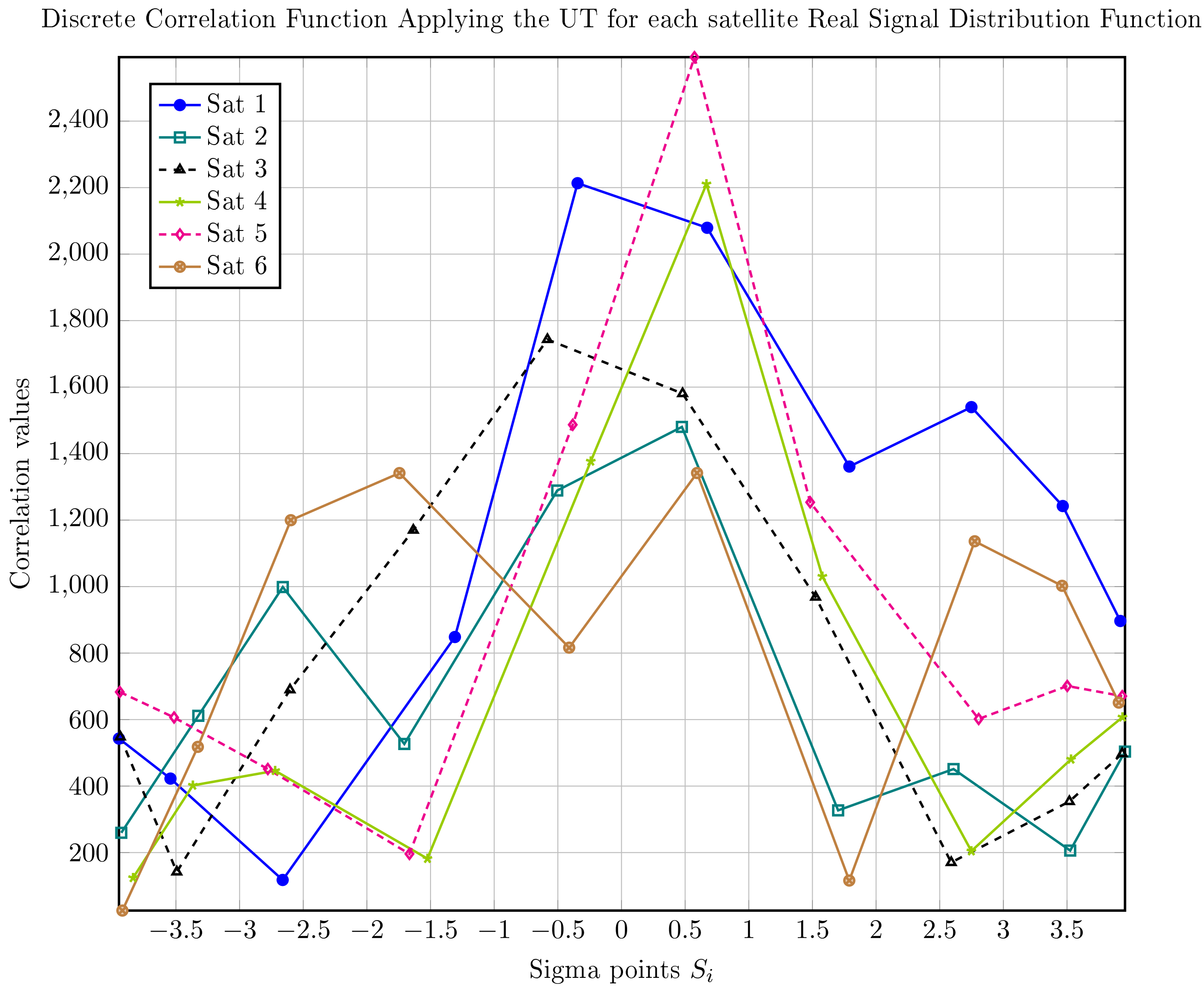

Figure 6.6: Discrete correlation curve of Figure 6.2 using sigma points originally from each satellite real signal distribution function.

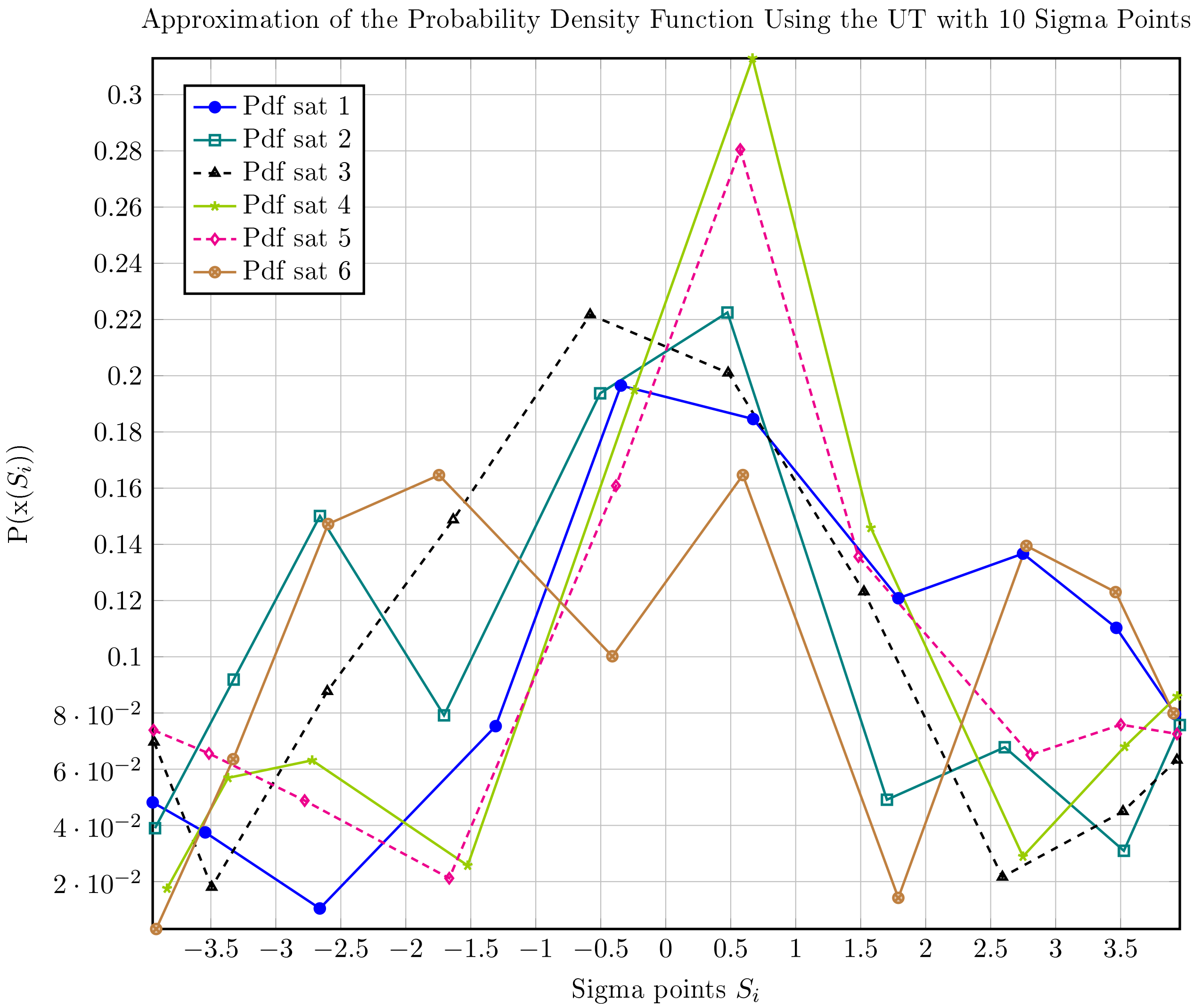

Figure 6.7: Discrete Probability Density Function using sigma points originally from each satellite real signal distribution function.

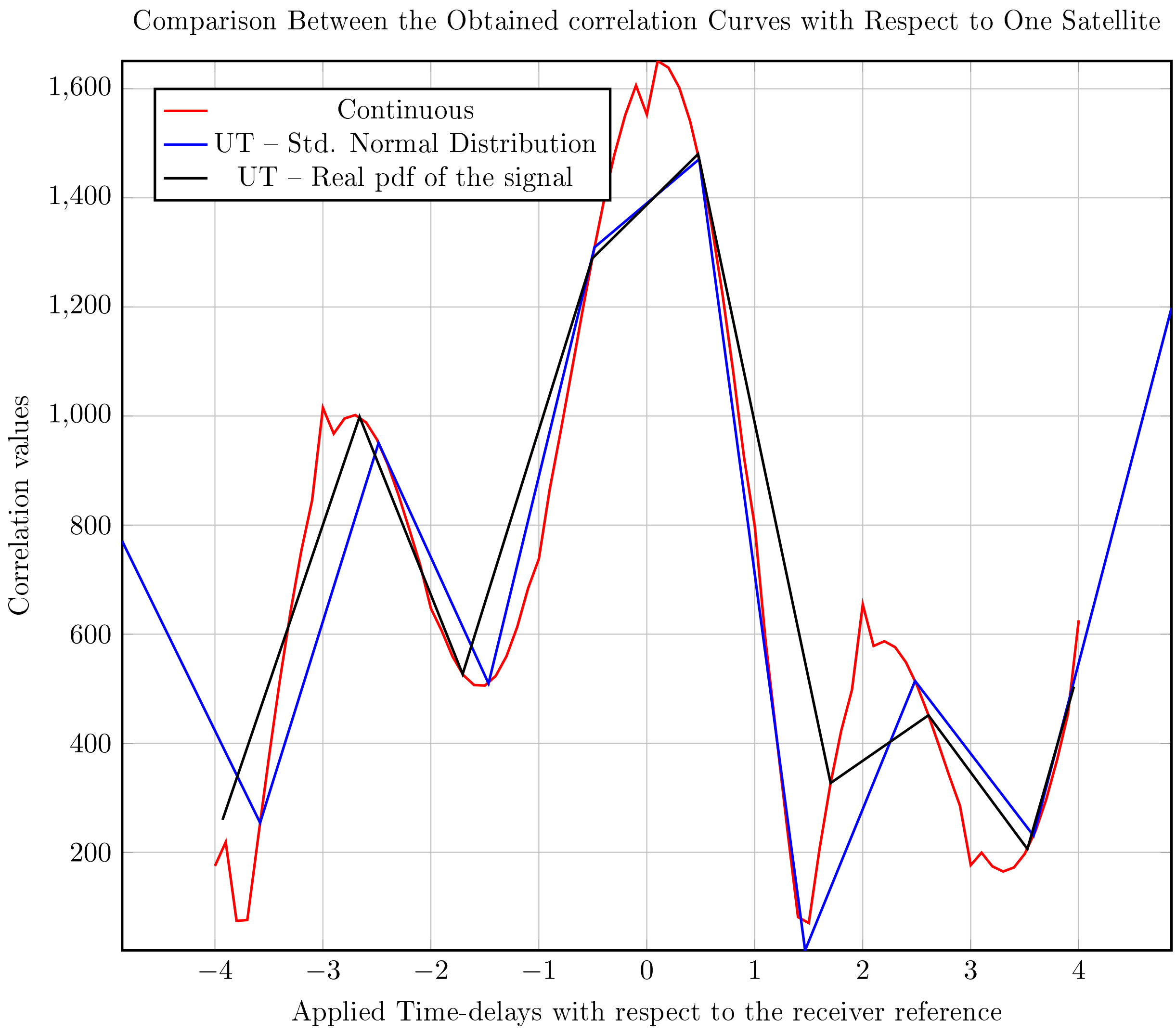

Figure 6.8: Correlation curves of only one satellite representing each case: Continuous, Standard Normal UT and UT with the Calculated Moments of the signal's real probability distribution function.

7 Refining the Time-Difference of Arrival Estimation

Chapter 6 presents the validity of the UT and shows that it legitimately conserves the statistical moments of the probability distribution through its calculated sigmas and weights. Table 6.1 pointed out the same values for the mean and variance of the original distribution and its discrete form after applying the UT.

Although the discrete curves obtained in Chapter 6 give a good estimation of where the signal time-delay most probably is, it turns out to be difficult to decide for one specific, since the peaks of the curves refers to intervals, not pointing to a specific point on the correlation function. Thus, this chapter brings the second application of the UT by using the sigmas from a delimited interval, that the results from the first UT application pointed out as being the most probable. The intention is to provide an even closer look at the distribution.

The results shown in this Chapter have also been obtained through running simulations in MATLAB. From the discrete curve calculated from the UT, the interval which contains the maximum correlation with one sigma gap in both directions from the peak has been separated and brought in its continuous form again. By doing so, the part where the probability to find the signal delay that matches to the transmitted signal is insignificant, is discarded. From this reduced interval in its continuous form the new PDF is calculated, as well as the new moments of the new distribution. The UT is then applied a second time, in order to reach an even closer discrete look to the TDOA estimation.

7.1 Applying the Unscented Transform to a Delimited Interval

The current section studies the use of the UT as a tool to give a better overview of what should be the best time interval containing the most probable delay of the GNSS signal.

From the curve of the correlation discretized by the UT, Figure 6.6, the area of interest is separated from the rest of the curve, and its correspondent interval in the continuous form is transformed by the UT. The result of this operation can be seen in Figure 7.1, where a zoom of the correlation around its peaks is obtained.

Once there is a delimited interval of the correlation function, the UT can be applied again and the sigmas and weights calculated, this time related to the reduced gap involving the region of the correlation peak.

One again the UT confirms its property of conservation of the statistical moments of the distribution. The first moments of the characterized curves in Figures 7.1 and 7.2 are exactly the same no matter how strong the noise is, which is fairly reasonable, since the UT points have been calculated taking the distribution in Figure 7.1 into consideration.

At the second application of the UT, the region with highest probability of having the true delay is taken aside (and later also discretized), and the rest of the original function is discarded. If comparing the moments between the original function and the one reduced to a small interval, both in its smooth shape, they assume different values. In fact, such behavior is expected, since a new distribution is generated at the moment the probability out of the zoom is rejected. That is why the UT has been used twice, because there are two functions with different moments.

Although basic receivers use three correlators to track the incoming signal, the number of correlators could be increased in the order to extract further information from the signal [11]. This kind of technique is actually applied in advanced signal processing receivers in order to achieve higher performances. Thus if the goal is to have an estimation with high definition, that is, with many points, it would be necessary to calculate a UT with many points as well. However, this Chapter shows that instead of calculating the UT with many sigma points, another possibility is to apply the UT more than once.

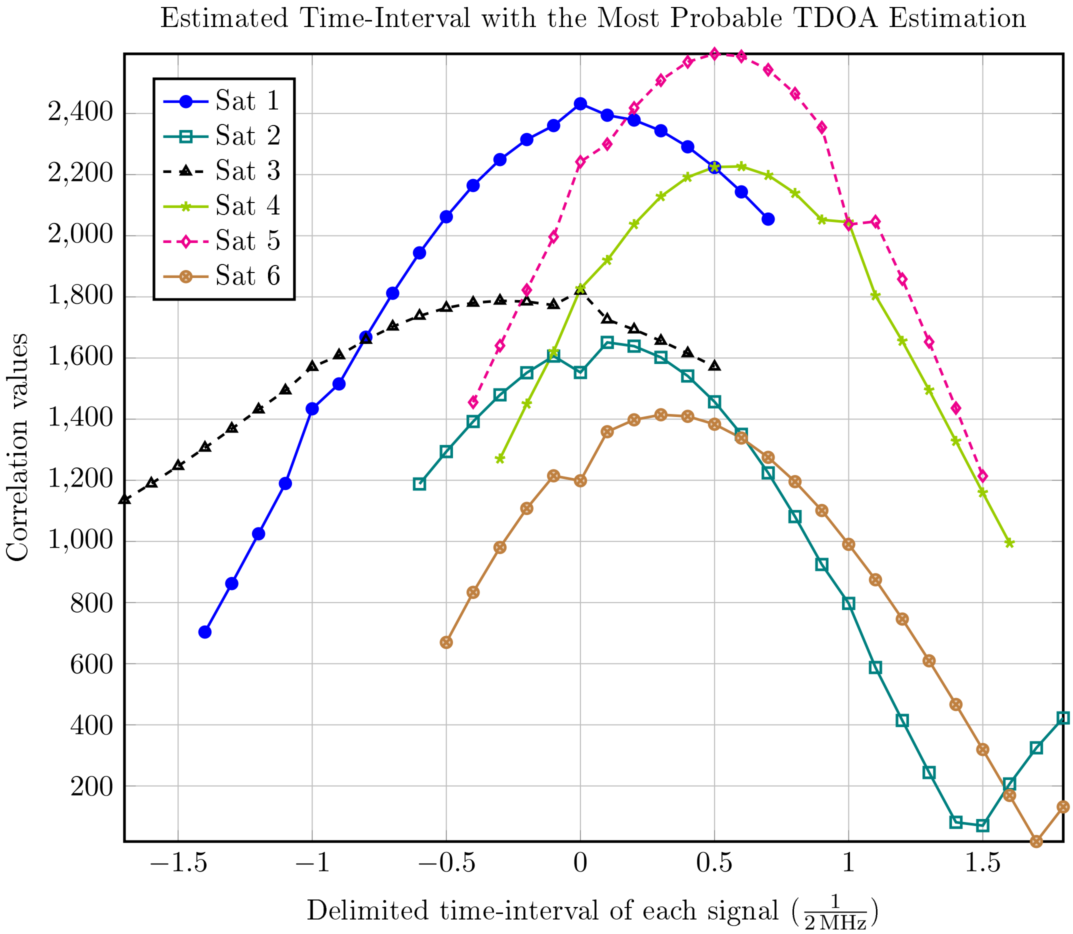

Figure 7.1: Correlation curves delimited to the peak interval, which has been pointed out by one first use of the UT, and then reproduced in the smoothly curves.

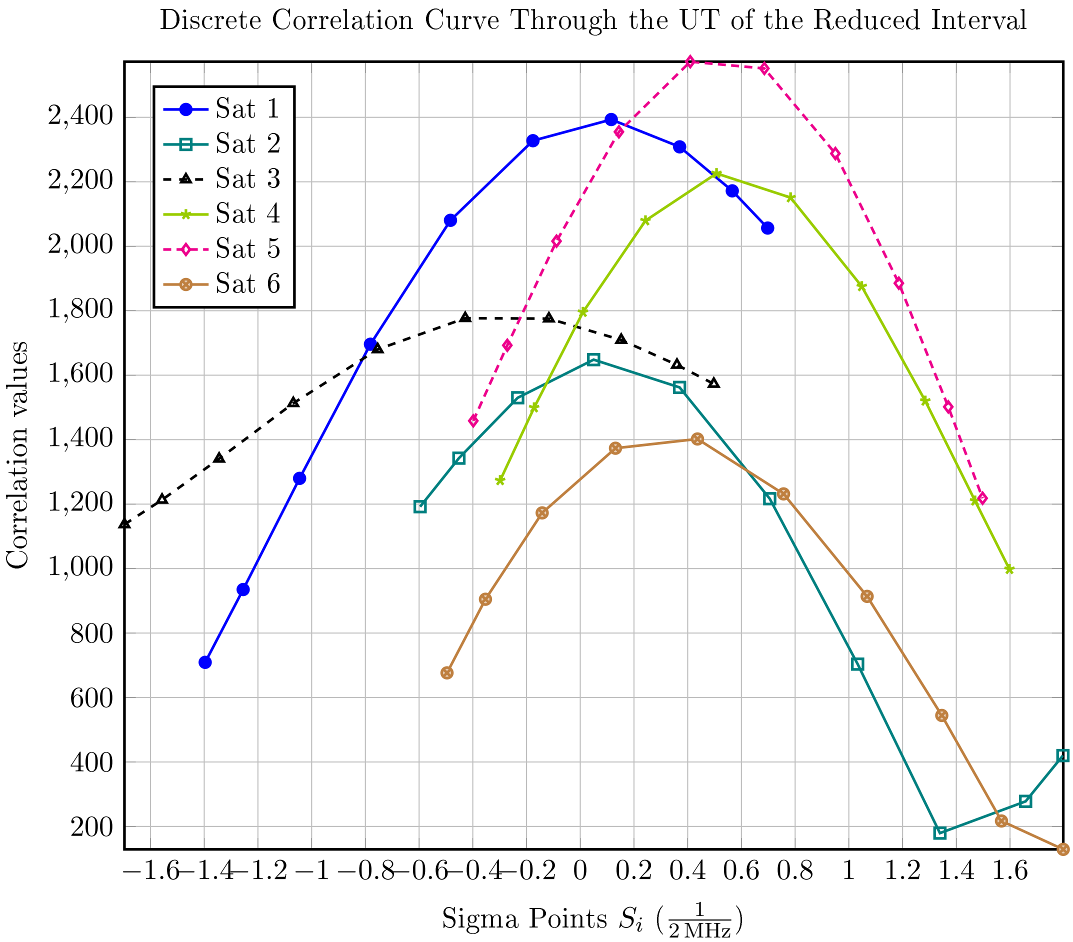

Figure 7.2: Discretized Correlation curves from Figure 7.1 obtained through the second application of the UT.

8 Achieved Results

Previously, in Chapters 6 and 7, the use of the UT in the TDOA estimation of a signal coming from a GNSS satellite (in this work, from the GPS system) was shown so far to be a trustful alternative to be carried out.

An early outcome from handling the approximation by means of the UT is the aimed discretization of a PDF accordingly to the desired number of sigma points. In Chapter 6 the UT confirms its property of conservating the statistical moments of a distribution, already anticipated in Chapter 5. Each signal distribution function presents its statistical moments, which are successfully conserved in the sigma points given through the UT application. From that, it becomes more evident that the range where the most probable signal time-delay is located.

In Chapter 6 the discrete approximation of the signal distribution has been done taking first the Standard Normal distribution into consideration, and later the signal's own PDF. As presented in Section 6.2, it was shown that assuming the Standard Normal Distribution, which follows $\mathcal{N}(0,\sigma^2)$ with zero mean and variance σ^2, there has to be no multipath signals influence. In such situations, the NLOS signal presents its mean value shifted from the reference established at the receiver. The value for the variance varies accordingly to the presence of noise, and despite the fact that sometimes the variance may not assume unitary value, the application of the UT expecting a Standard Normal function distribution, in case of no multipath, would still provide good results.

In Section 6.3 the UT is applied following the real signal PDF, the one at the time of arrival. Indeed, the situation is more specific and completely keeps the moments of the original distribution in its discrete approximation. The Table 6.1 confirms it, by showing the same moments found for the continuous signal correlation function and its discrete version calculated by the UT. When considering the signal own PDF the presence of noise and also of NLOS signals can be accepted not influencing the application of the algorithm.

The next step comes as the reduction of the time-interval to be worked on, as presented in Chapter 7. The goal is to obtain a more detailed estimation of the time-shift delay of the incoming signal. In order to carry it out, the region near to the peak of the function, which also contains the peak, is separated from the rest of the original function and the part not included in the peak-interval is discarded, since it is not relevant to the time-delay estimation.

From the interval reduction, a new distribution with different moments is obtained, followed by application of the UT for a second time. The use of the UT in the new function provides a more accurate answer in the TDOA estimation.

9 Conclusion

This work presents a new use of the already known Unscented Transform. Its application in the TDOA estimation shows another possible way of discretization that keeps a good accuracy, without generalizing or approximating data randomly as it has been done.

In fact, the spread sprectrum signal coming from the satellite after having its spectrum-spreading modulation removed, is fed to the baseband processing block. At this component, there are channels, each assigned to a specific SVID code. As part of the process in a generic GNSS receiver, inside the channels, during the acquisition phase, it is tried to find a visible GNSS signal. Once the signal has been found, starts the tracking mode, which by means of synchronization to the known PRN code, builds the process around the principle of signal correlation.

The correlation principle is first used to search for the satellites in view. Then, while in the acquisition mode, several correlations between the incoming (and also already identified) signal and multiple replicas of the possible PRN expected code are made, each replica generated for different code delays and Doppler frequency. When the local replicas and the incoming signal are aligned, their correlation generates a sharp format, where the code delay and Doppler frequency correspondent pair is. This outcome is then assumed to be a good estimate to begin the tracking process.

The analog signal transmitted from the satellite is digitalized at the receiver. The correlation that is currently used is performed on arbitrarily chosen points. That may result in inaccurate position information, as the estimation of the TDOA is based on a few arbitrary discrete points of the signal correlation function.

Thus, this work has proposed another way to obtain the discrete signal correlation function, by applying the UT. The UT is a transformation that preserves the statistical moments of the signal delay probability distribution. Therefore, the discrete correlation function of the signal is not arbitrarily achieved, but rather performed on points that truly maintain traces from its original continuous format. Thus, the proposed discrete form from the use of the UT may contribute with a faster and more accurate user position estimation in subsequent applications.

9.1 Future Perspectives

The proposed tool in this work can be tested in eventual future works, in order to make a direct comparison between the actual mechanism and the one that uses the UT. An interesting parameter to compare would be the effective time necessary to process the signal, as well as if the obtained answer is actually more accurate.

Aware that discrete functions makes the whole process easier, by an enormous economy of storage in keeping data and/or by building a decision, studies on finding new ways to come to a reasonable result should continue and may bring improvements to the methods already used in signal processing.

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