Universidade de Brasília – UnB
Faculdade UnB Gama – FGA
Aerospace Engineering

Application of the Background Oriented Schlieren Technique for Quantitative Measurements of Transonic Flows

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Brasília, DF
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Dedication

This work is dedicated to my family and friends, who never hesitated to help me through this process and keep me motivated.

Acknowledgements

First I would like to thank my family for all the support and sacrifices they made to provide me with the best education possible. I am also very grateful to my fiànce Christina for all the encouragement and love.

This thesis would not have been possible without the opportunity given to me to intern at the *Institute of Aviation of Warsaw* in Poland, where this work was developed. More specifically, I want to thank Wit Stryczniewicz for all the insight and knowledge I learned from him.

In addition, I would like to thank Professor Shynkarenko for all the insight and mentoring during my last two years of college.

Abstract

Flow visualization techniques have progressed from methods based on qualitative description of flow-field into techniques providing both qualitative and quantitative results. In this work, we present the implementation of Background Oriented Schlieren (BOS) technique for measurements of density values at transonic speeds. The density values obtained by BOS were then compared with results from Computational Fluid Dynamics (CFD) analysis of the same flow characteristics. In both qualitative and quantitative terms, the BOS and CFD results agreed, which proves the applicability of BOS for transonic flows.

Key-words: Background Oriented Schlieren. Transonic Flow. Flow Visualization.

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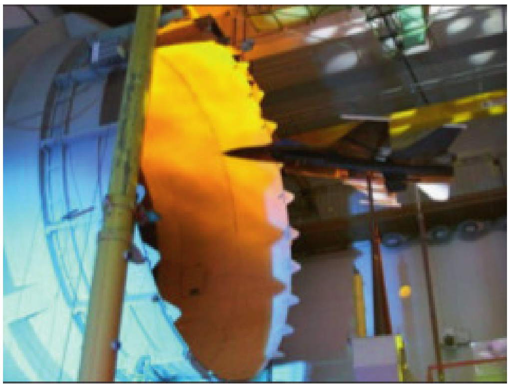
1 Introduction

In the early development of aeronautical science, human kind attempted flight in many ways, most of them unsuccessful. One clear example are the attempts of using birdlike wings to fly. All these failures occurred due to poor understanding of how aerodynamic forces such as lift, drag work, and their impact on stability. Inventors then started to perform on-ground tests previous to flight in order to investigate aerodynamic characteristics of their inventions. In the early days of flight development, the on-ground tests were performed mostly with adapted fans that replicated airflow around a body. These tests lead to the development of wind tunnels as we know them today, which are crucial for aerospace design. Currently, the general order for aircraft design is "tunnels test first, free-flight tests later" (BAALS; CORLISS, 1981).

Wind tunnels have developed from very simple fan setups to modern devices that can achieve high Mach numbers and accurately measure flow properties. Figure 1 shows the low speed wind tunnel at the *Institute of Aviation of Warsaw*. It consists in a closed-loop tunnel, with an open measuring space, which has 5 meters in diameter and is 6.5 meters long (ILOT, 2015).



(a)

(b)

Figure 1 – Low Speed Wind Tunnel at the Institute of Aviation of Warsaw (ILOT, 2015).

The wind tunnel development was accompanied by the advent of high speed flight and the problems associated with it. Advances in computational power and image digital processing allowed the rising of optical methods, which are capable of visualizing high speed flows (Fig. 2). The main optical methods are: *Shadow*, *Schilieren* and *Interferometry* (RISTIC, 2006). Optical methods have the great advantage of providing whole flow-field visualization and do not require the usage of intrusive instruments such as probes, except for calibration purposes.

These methods are an attractive alternative in the analysis of complex flow-fields, where big gradients are present, such as transonic flows. Figure 2 displays the usage of schlieren technique to visualize shock and expansion waves around a body under supersonic conditions.

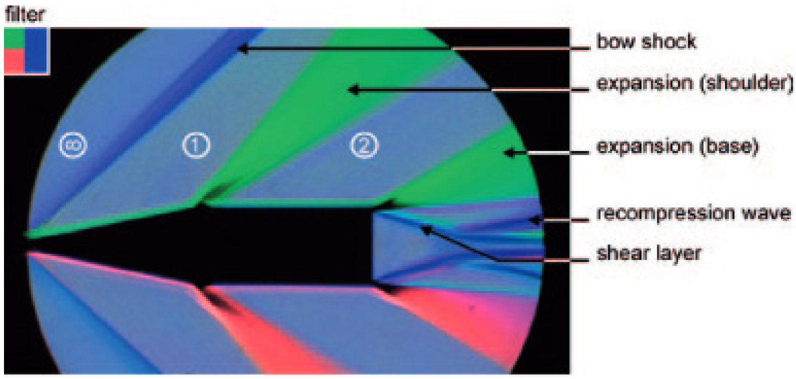


Figure 2 – Color schlieren image of supersonic flow around a 2D wedge-plate model (ELSINGA et al., 2004).

Although optical methods are apparently able to provide only qualitative aspects of the flow, advances in computational capability and image processing allowed these techniques to also provide quantitative aspects of the flow. As an example, the *Background Oriented Schlieren* (BOS) (RAFFEL, 2015) is capable of computing density values of a given flow-field through image processing. This technique is based on variation of light's refraction index due to density gradients, which are mathematically related. The BOS technique and its principles are further explained in section 2.2.

1.1 General Objectives

In this work, the implementation of *Background Oriented Schlieren* technique for measurements of the density gradients at transonic speeds is presented. Analysis on a conical body submitted to transonic flow of Mach 0.9 inside a wind tunnel test section is performed. In the past decade, the BOS technique has been applied for measurements of compressible flows in wind tunnels, supersonic jets (RICHARD et al., 2002) and full-scale measurements. Although, the possibility of using density measurements with BOS technique at supersonic speeds (VENKATAKRISHNAN; MEIER, 2004) has been confirmed, the feasibility of BOS for density field reconstruction at transonic flow regime has not been explored.

1.2 Specific Objectives

The aim of the presented research was to perform BOS flow visualization and develop a procedure for determination of density field at transonic speeds. The results obtained by this procedure are then validated by *Computational Fluid Dynamics* (CFD) simulations that numerically determine the density field around the conical body.

In this work, development of BOS measurement setup as well as data reduction procedures are presented. A dedicated procedure was developed for retrieving the density field from the experimental data. The presented methodology can be used for investigations of compressible flows at transonic speeds as well as can be extended for supersonic flows.

2 Physical and Mathematical Fundamentals

2.1 Transonic Flows

The analysis of transonic flow became important as flying devices were able to reach higher speeds. During the second world war airplanes started reaching transonic speeds (typically Mach number from 0.8 to 1.2) (ANDERSON, 2010) when diving. Major problems related to shock waves arose, preventing airplanes from recovering from dives. The shock waves formed at transonic speeds lead to a rapid increase in drag due to wave drag and because of pressure rising through a shock wave. Therefore, transonic flows are a main concern for aircraft designers.

The physical behavior of transonic flow can be challenging and difficult to predict. This happens because mixed sub- and supersonic flow occurs in the same flow field at transonic speed (Fig.3).

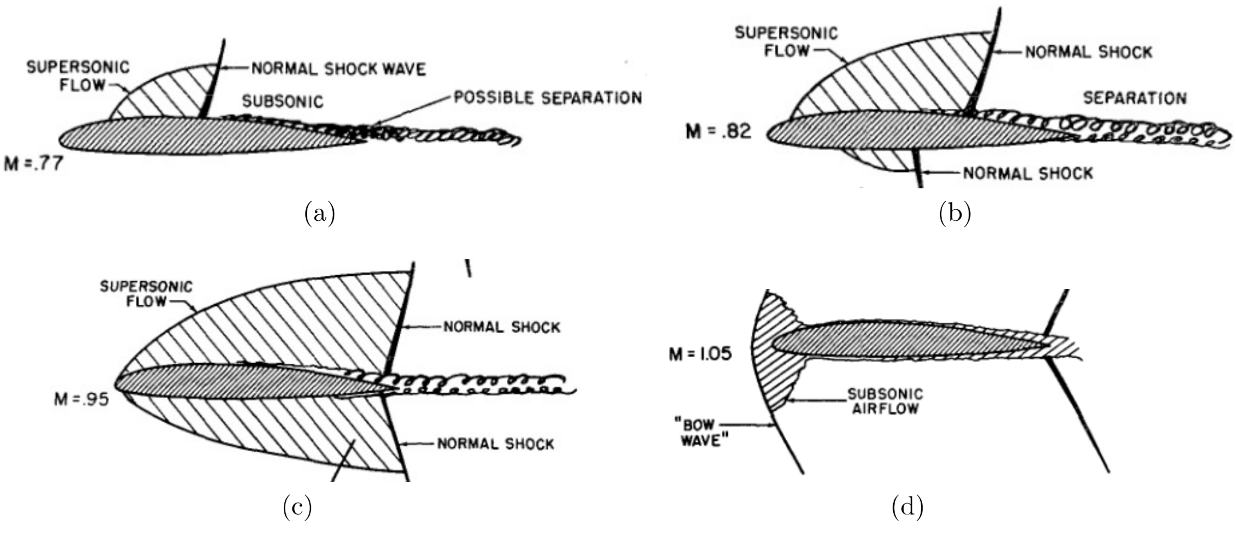
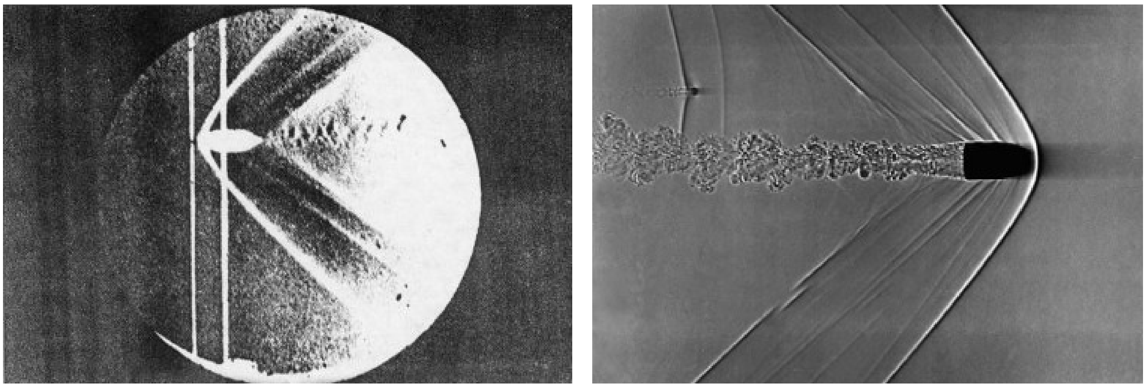


Figure 3 – Development of transonic flow around an airfoil as Mach number increases (HURT, 2013).

Figure 3 displays an airfoil at different transonic speeds. As Mach number increases, a supersonic flow region develops, and the flow is brought to subsonic regime by the occurrence of a normal shock wave. As Mach number approaches sonic speed, the shock waves move to the trailing edge (Fig.3c) until Mach finally becomes slightly greater than one. When this happens, the shock waves become oblique (Fig.3d). This rapid variation of flow pattern leads to an equally rapid increase in drag, lift and pitching moment.

Transonic flows are also difficult in terms of mathematical modeling. That type of flow is inherently non-linear, and the steady solution changes math types, being elliptic in the subsonic portion of the flow and hyperbolic in the supersonic parts (MASON, 2018). The lack of an analytical method that represents transonic flows lead to the development of computational and flow visualization methods. A computational method for transonic flows was developed by (MURMAN et al., 1970) using small disturbances theory. Flow visualization techniques are widely used in qualitative analysis of transonic flows, since they allow the observation of the location of shock waves.

These techniques have been applied to transonic flows since the 19th century, when Ernst Mach and his son used schlieren photography to visualize the shadows of the invisible shock waves of the flow pattern of a bullet (Fig.4a). Flow visualization techniques have greatly progressed due to advancements in photographic devices, digital image processing and computational capabilities (Fig.4b). Currently, some flow visualization methods are even able to perform not only qualitative analysis of the flow, but also provide quantitative flow information.



(a) (b)

Figure 4 – (a) Photograph of a bullet in supersonic flight, published by Ernst Mach in 1887 (ANDERSON, 2003). (b) Shadowgraph depicting the flow generated by a bullet at supersonic speeds (DAVIDHAZY, 2014).

Transonic flows are indeed a complex phenomena that have to be taken into account in aircraft design. Given that the there is very little analytical knowledge available on transonic regime, flow visualization techniques are very attractive alternatives in the analysis of such flows.

2.2 The Background Oriented Schlieren Technique

The Background Oriented Schlieren technique is based on the relationship between refraction index of light and density gradients. These gradients cause light beams to deflect as show in Fig. 5. This phenomenon is similar to observation of heat haze or mirage, where local density gradients between the human observer and a distant object distort the perceived image (RAFFEL, 2015). Density gradient zones in aeronautical applications could be shock or expansion waves, high temperature zones or the presence of flow control devices.

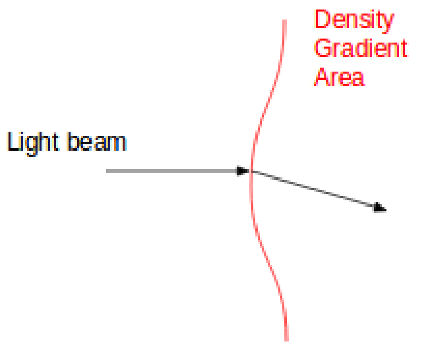


Figure 5 – Light deflection due to density gradients.

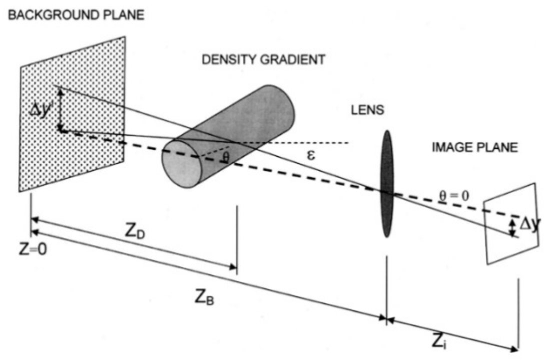


Figure 6 – BOS setup. (RAFFEL, 2015)

One of the goals during BOS analysis is to quantify the displacements caused by density gradients. In order to do so, images are taken of the area of interest, as for example the test section of a wind tunnel. Two types of images are taken, the first one with flow at rest (wind-off) and another one with flow on operation (wind-on). The images will be displaced when compared due to the deflection of light caused by density gradients at wind-on conditions. Figure 6 shows schematics for BOS created by (VENKATAKRISHNAN; MEIER, 2004). This figure contains the *density gradient* represented by a cylinder, a *background* behind the gradient area, the *lens* in which the light rays go through to form an image at the *image plane*. The dotted line represents a light ray correspondent to a point on the *background* passing through the test section with no deflection (flow at rest). Once flow is on operation, the same light ray is deflected by an angle ε due to density gradient, and it forms an image displaced by Δx and Δy. Once both wind-off and wind-on pictures are obtained, the *PIV Adaptive Correlation Algorithm* (RAFFEL et al., 2007) is used to quantify the displacements.

The displacements (Δx and Δy) caused by the gradient zone are associated with the local refraction of index n, which can be mathematically related to flow density ρ by the Gladstone-Dale equation (Eq. 2.1). The constant $G(\lambda)$, in Eq. 2.1, is called the Gladstone-Dale constant. It depends on characteristics of the gaseous media and also on the frequency of light used. Therefore, the focus of BOS algorithm consists in finding refraction index of every point within the flow-field, which directly leads to density field as stated in Eq. 2.1.

$$\frac{n-1}{\rho} = G(\lambda) \tag{2.1}$$

The refraction index n is related to the obtained image displacements, (Δx and Δy), by the partial differential equation represented in Eq.2.2 (ORTIZ et al., 2016). The constants n_0, M, Z_d and h in equation 2.2 represent refractive index of undisturbed flow, optical magnification, distance between background and schilieren object and object thickness respectively. Once this equation is integrated (see section 2.3), an the refraction indexes for each point are found, equation 2.1 can be used to obtain density for each point. This concludes the final purpose of this technique of quantitatively finding the density field.

$$\frac{\partial^2 n}{\partial x^2} + \frac{\partial^2 n}{\partial y^2} = \frac{n_0}{M Z_d h}\left(\frac{\partial \Delta x}{\partial x} + \frac{\partial \Delta y}{\partial y}\right) \tag{2.2}$$

This visualization technique is therefore not only capable of extracting qualitative information about the flow-field but also quantitatively extract density values. This makes BOS an attractive method for analyzing complex flow types, especially flows at transonic speeds.

2.3 Finite Element Difference Applied to BOS

The finite difference method was applied to the BOS technique in order to solve equation 2.2. An in-house code was developed to solve the partial differential equation (Eq.2.2) responsible for computing the refraction index n. The pixels of images acquired were treated as single nodes as shown in Fig.7.

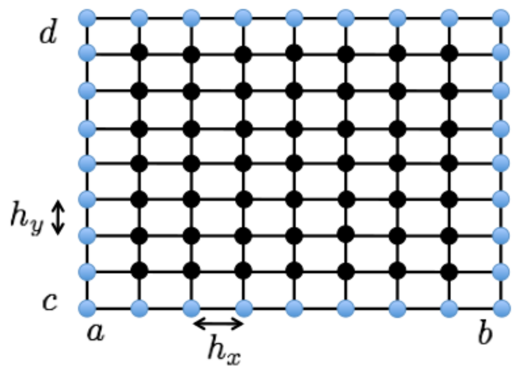


Figure 7 – FEM applied to BOS images.

The 5 point difference was applied in order to perform discretization of Eq.2.2. One central node (pixel), is surrounded by four other points as shown in Fig. 8. The partial derivatives can then be represented by Eqs. 2.3 and 2.4, where the therm $u_{i,k}$ represents the central node.

$$\frac{\partial^2 u}{\partial x^2} = \frac{u_{i-1,k} - 2u_{i,k} + u_{i+1,k}}{h_x} \tag{2.3}$$

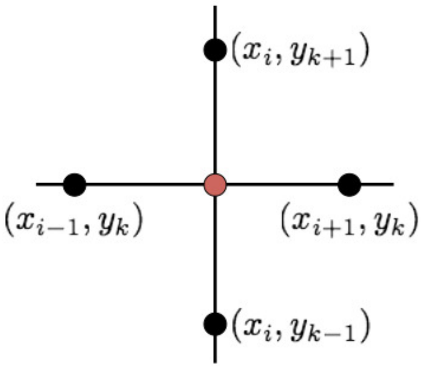


Figure 8 – FEM applied to BOS images.

$$\frac{\partial^2 u}{\partial y^2} = \frac{u_{i,k-1} - 2u_{i,k} + u_{i,k+1}}{h_y} \tag{2.4}$$

Substituting these values on Eq.2.2, an algebraic equation is formed as shown in Eq.2.5.

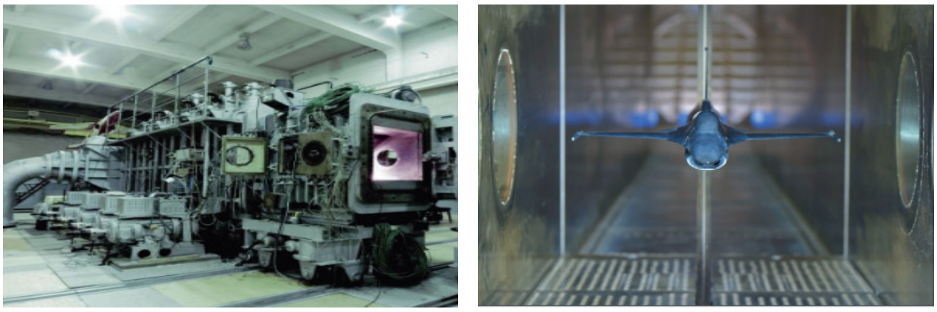
$$\frac{u_{i-1,k} - 2u_{i,k} + u_{i+1,k}}{h_x} + \frac{u_{i,k-1} - 2u_{i,k} + u_{i,k+1}}{h_y} = f_{i,k} \tag{2.5}$$

The therm $f_{i,k}$ represents the right hand side of Eq. 2.2. Once the discretization is performed, the equation is solved using successive over relaxation. The solution provides then the refraction index n which necessary to compute density.

3 Methodology

3.1 Experimental Apparatus

The BOS experiments were carried out at the Trisonic Wind Tunnel of the *Institute of Aviation* in Warsaw, Poland. The wind tunnel and its specification are show in Fig.9. The tunnel is a blow-down type with partial recirculation of the flow and can be operated in subsonic, transonic and supersonic regimes. Operation Mach numbers range from 0.2 to 2.3 with accuracy ±0.01.



(a) (b)

Main features	
Test section dimensions:	
height x width	0.6 x 0.6 m
length	1.5 m
Test gas	air
Wind speed	M = 0.2 - 2.3

(c)

Figure 9 – High Speed Wind Tunnel at the Institute of Aviation of Warsaw (ILOT, 2015).

A conical body is placed inside the wind tunnel test section and submitted to an airflow of Mach number 0.9 as demonstrated in Fig.10. The BOS setup is then used to obtain the density field around the conical body.

The Background Oriented Schlieren is based on digital analysis of image displacements. The background pattern is illuminated by source of light providing constant level intensity over the whole test time and imaged through a fluid containing spatial density gradients (Fig. 11). Since coherent beam of light is not required, the BOS setup is much simpler than classical Toepler schlieren or shadowgraph schlieren apparatus.

The images were captured with Nikon d800 digital SLR camera with 36 MP CMOS sensor and Nikon Nikkor AF-S 70-200 mm f2.8 lenses. The focal length was set to 200

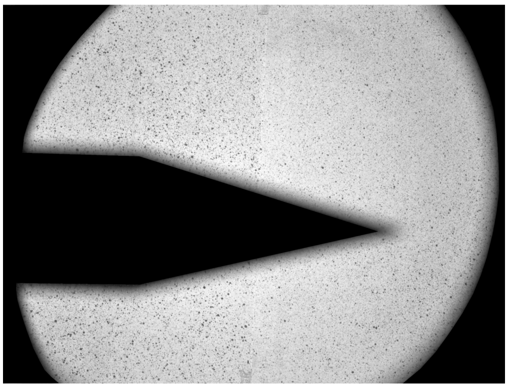


Figure 10 – Conical body inside the Trisonic Wind Tunnel Test section.

mm. The background was created by spraying black paint on white paper. A series of images was created by varying the pressure in the airbrush. The series varied from very small (1-2 mm in diameter) and dense spots to big (5-8 mm) and sparse dots. The optimal density, providing images of dots size from 1 to 3 pixels on wind-off images, was chosen from the series of background. In order to provide stable illumination of the background, a HardSoft IL-106X pulsed LED illuminator (STASICKI et al., 2010) with a green LED was utilized. During the test, the illuminator was in continuous work mode and the images were acquired by remote shutter release.

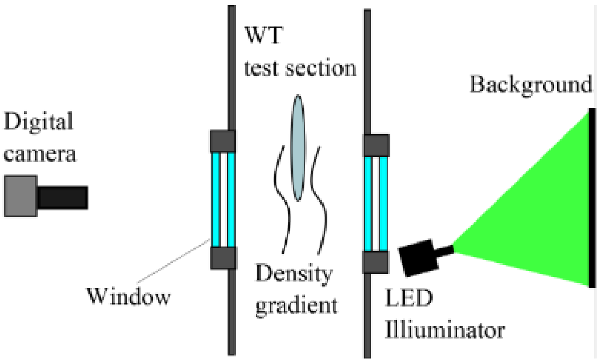


Figure 11 – Experimental setup.

3.2 Computational Processing

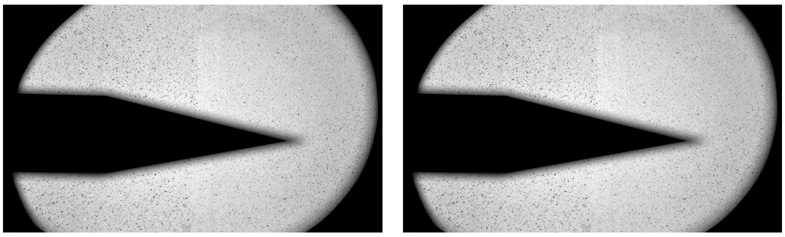
The Background Oriented Schilieren technique can be summarized in the following steps:

(i) Acquisition of reference image (*wind-off*) of background without flow and images during the wind tunnel run (*wind-on*).

(ii) Determination of the apparent image displacements between the wind-off and wind-on images by digital image processing.

(iii) Integrate Poisson equation (equation 2.2) in order to obtain the density field.

The images acquired for both *wind-off* and *wind-on* conditions are displayed in figure 12.

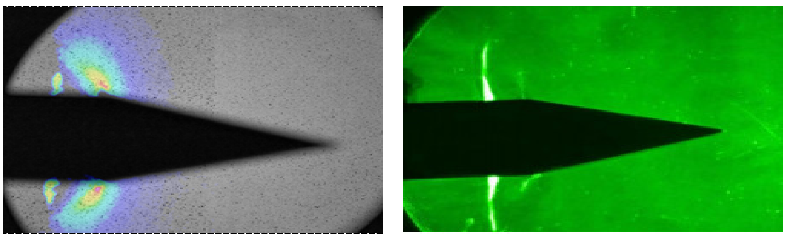


(a) (b)

Figure 12 – Images acquired of the wind tunnel test section: (a) Wind-off. (b) Wind-on.

There is no apparent difference between wind-off and wind-on images as one can easily notice in figure 12. However, when both images are represented by matrices of pixel values, the differences become clear. Based on that, the *PIV Cross-Correlation Analysis*(STRYCZNIEWICZ, 2012) computes the difference of both images pixel by pixel. The maximum difference corresponds to the most probable displacement, and generates a corresponding vector in each interrogation area of the image.

Adaptive PIV algorithm (THEUNISSEN; SCARANO; RIETHMULLER, 2007) was applied to both images, where the areas with no displacement (body area) was removed. The resultant displacement field is shown in figure 13. One can easily notice that the greater displacements are concentrated in the regions where expansion and shock waves appear (greater density gradients).

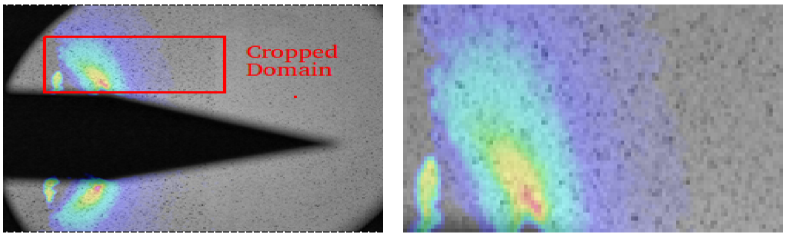


(a) (b)

Figure 13 – Schlieren images of conical body.

Once the displacement field is obtained, we have all the information required to integrate equation 2.2. In order to restrict our analysis to the flow area of interest, we crop the domain as demonstrated in figure 14a.

Equation 2.2 is then integrated over the cropped domain using the finite element method five point difference with successive over relaxation (MATHEWS; FINK, 1999).



(a) (b)

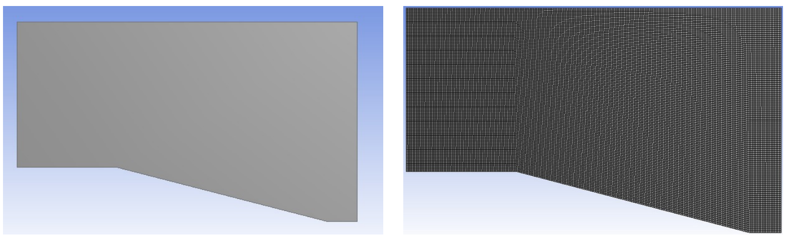
Figure 14 – (a) Area where domain is cropped. (b) Cropped domain.

Dirichilet and Neumann boundary conditions are applied to the edges parallel and perpendicular to the flow respectively (ORTIZ et al., 2016). The Dirichilet boundaries were set to be equal the undisturbed flow refractive index ($n = n_0$) and the Neumann boundaries were set to zero. Once the refractive index field was obtained, density was calculated using the *Gladstone-Dale* equation (Eq.2.1).

A *Computational Fluid Dynamics* (CFD) simulation was also performed using the commercial code *Ansys Fluent* in order to check density values feasibility. *Fluent* is a *Pressure based* solver, where the governing equations are solved sequentially (segregated from one another).

The simulation domain and the mesh are represented in figure 15a and 15b. A total of 26400 elements were used. The boundary conditions were set as follows:

- Farfield of Mach 0.9 on the right, top and left edges.
- Physical walls on the diagonal edge and the adjacent to it.
- Symmetry line ahead of the conical body.



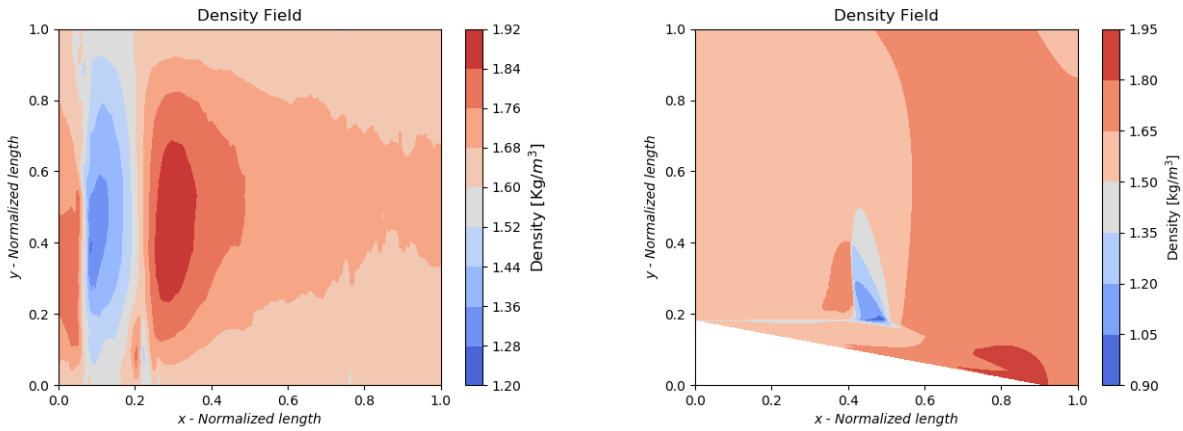
(a) (b)

Figure 15 – (a) Simulation domain. (b) Simulation mesh.

4 Results and Discussion

In this section, the outputs obtained from the BOS algorithm are discussed and compared to CFD results in both qualitative and quantitative terms. It is important to notice that the BOS domain is equivalent to cropping the CFD geometry in the regions of greater gradients.

The density field is presented in figure 16 for both BOS algorithm and CFD analysis respectively. One can notice in both figures the local presence of expansion and shock waves, which is a strong characteristic of transonic flows. The fact that BOS displays a similar flow behavior when compared to CFD proves that this technique is able to achieve good results in qualitative terms. It can also be used to determine space location of flow phenomena such as shock and expansion waves.

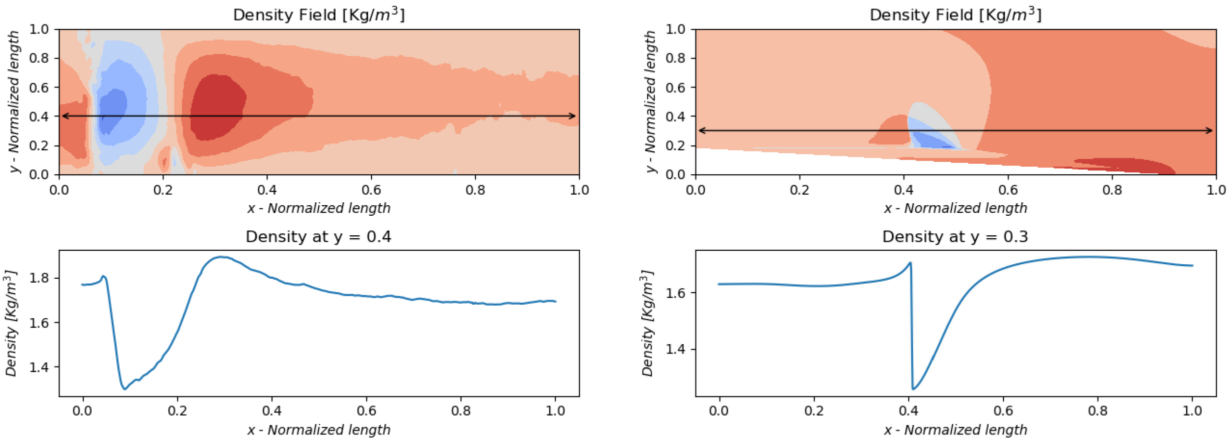


(a) (b)

Figure 16 – Density fields obtained by (a) BOS. (b) CFD.

In order to check density values, a line was traced across the flow domain for both BOS and CFD analysis in such a way that it would pass through the greater gradient regions as displayed in figures 17a and 17b. These lines are responsible for extracting density values from each data point. The density lines show great similarity between BOS and CFD results, as well as almost the same behavior in terms of increase and decrease of density. Table 1 compares the maximum, minimum and average values extracted by the two lines as well as the difference between them.

Comparing the numerical values shows that the BOS outputs match the CFD results within a small error range. This technique is therefore capable of not only displaying qualitative characteristics of the flow, but also quantitatively calculating density values. BOS is indeed a proof that flow visualization techniques are powerful tools that can measure flow properties.



(a) (b)

Figure 17 – Density lines obtained by (a) BOS. (b) CFD.

Table 1 – Density lines comparison.

-	Density (BOS)	Density (CFD)	Difference
Maximum Value [Kg/m^3]	1.89	1.72	9.8 %
Minimum Value [Kg/m^3]	1.29	1.25	3.2 %
Average Value [Kg/m^3]	1.69	1.64	3.0 %

5 Conclusion

Flow visualization techniques have greatly progressed from methods of qualitative description to methods capable of accurately measuring flow properties.

The results demonstrated by figures 16 and 17 show that the BOS technique was able to match CFD results in qualitative terms. Table 1 also demonstrates that the density values for both BOS and CFD agree, which validates BOS as a quantitative technique.

Background Oriented Schilieren together with the *PIV Correlation* technique are capable of not only providing qualitative information about the flow, but also compute the density flow-field within a small error range. This technique also proved to be very convenient for the analysis of transonic flows, and can be further applied to more complex types of flow. The simplicity of the BOS setup makes it attractive to use for comparison of the CFD results with Wind Tunnel experimental results for verification of the numerical simulations results.

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